MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXX. JUNE 11, 1920. No. 8

Professor A. FOWLER, F.R.S., PRESIDENT, in the Chair.

Captain George Frederick Dickens, D.S.C., Lieut. R.N.R., 24 Southside, Weston-super-Mare;
Walter Gornold, 3 Medina Villas, Hove, Sussex;
R. Leetham Jones, M.A. (Cantab.), 11 Eardley Crescent, Earl's Court, London, S.W. 5;
Gerald Merton, M.A. (Cantab.), M.C., Slopes, Woldingham, Surrey;
J. de Mendizábal Tamborrel, Ingénieur Géographe, Sociedad Alzate, Mexico D.F., Mexico; and
Joseph Proudman, M.A., D.Sc., Professor of Applied Mathematics and Director of the Tidal Institute, University of Liverpool; and 32 Hawarden Avenue, Sefton Park, Liverpool,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Thomas Allison, Barrister, and Solicitor of the Supreme Court of New Zealand, 54 Glasgow Street, Wanganui, N.Z. (proposed by C. E. Adams); and
William Henry Connell, Engineers' Manager, T. Cooke & Sons, York, and Torver House, Stockton Lane, York (proposed by Wm. Shackleton).

Fifty presents were announced as having been received since the last Meeting, including, amongst others:—

Ernest W. Brown, Tables of the Motion of the Moon, 3 volumes, presented by the Yale University.

On the Systematic Motions of Stars derived from Radial Velocities. By J. Halm, Ph.D. (Plate 12.)

Modern investigations of the transverse proper motions of the stars have led to the definite conclusion that these motions, when considered relatively to the centroid of the stellar universe in its entirety, are not distributed according to mere chance. From the proper motions of the Auwers-Bradley Catalogue Kapteyn derived distinct evidence of a *systematic* arrangement of star motion, already indicated in the results of a previous work by Kobold, which led him to formulate his well-known hypothesis of *two star drifts.* Kapteyn's general conclusions were corroborated by subsequent investigations, especially those of Eddington and Dyson.

On closer examination it became evident, however, that though the two-drift hypothesis satisfied the observed features of stellar motion better than the older theory of random motions, certain well-marked peculiarities in the observed distribution remained unaccounted for. In a paper in *M.N.*, 71, 610, I pointed out that the discrepancies between the two-drift hypothesis and observation could be smoothed out to a considerable extent by the assumption of a third drift designated as Drift O. This additional drift was found to comprise stars whose individual motions relatively to the stellar centroid are distributed at random in all directions. Drift O therefore possesses no *systematic* motion in relation to the stars in general. A characteristic feature of the third drift was found in the fact that it includes all the stars of the spectral type B.

Broadly speaking, the three-drift hypothesis represents a combination of the older random conception with Kapteyn's two-drift hypothesis. Part of the universe (about one-third of the stars contained in our catalogues) consists of stars whose motions, after deducting the parallactic motion due to the transference of our solar system through space, are distributed at random in all directions. Superadded on this "chaotic" assemblage, two "drifts" are supposed to move as distinct and separate systems in opposite directions. The motions of the drift-stars consist of a systematic motion common to all the members of the drift and of individual motions relatively to the centroid of the drift, these latter being distributed at random in all directions.

The salient features of the original random or chaotic theory, the two-drift and three-drift hypotheses, are shown in their simplest form when motion in one co-ordinate only is taken into consideration, such as is represented in the radial velocities of the stars, for instance.

Let it be supposed that in any specified region of the sky the component of the sun's motion relatively to the stars in general has been deducted from the observed radial velocities, and let the distribution of the residual radial motions (*i.e.* the numbers of stars whose velocities lie between 0 and 1, 1 and 2, etc., kms. per second) be examined.

In the older random hypothesis it is assumed that these numbers are distributed in accordance with the law of errors. If N represents the total number of stars, the number whose velocities lie between v_1 and v_2 is expressed by the error integral:

$$N\frac{h}{\sqrt{\pi}}\int_{v_1}^{v_2} e^{-h^2v^2}dv \quad . \quad . \quad . \quad . \quad (1)$$

The random hypothesis is characterised by the condition that the modulus h has the *same* value in all regions of the sky.

In the two-drift hypothesis, as suggested by Kapteyn and Eddington, two separate error functions are required. Let N_1 be the number of stars in Drift I., N_2 the number in Drift II. Further, let Drift I. possess a systematic motion V_1 in the direction of the "vertex," Drift II. a motion of the amount V_2 in the opposite direction, and let λ be the arc of the great circle between the vertex and the region under consideration. The number of stars whose radial velocities lie between the specified limits v_1 and v_2 is expressed by

$$N_1\frac{h_1}{\sqrt{\pi}}\int_{v_1}^{v_2} e^{-h_1^2(v-V_1\cos\lambda)^2}dv + N_2\frac{h_2}{\sqrt{\pi}}\int_{v_1}^{v_2} e^{-h_2^2(v+V_2\cos\lambda)^2}dv, \quad (2)$$

$$N_1+N_2=N.$$

In the three-drift hypothesis a third integral of the form

$$N_0\frac{h_0}{\sqrt{\pi}}\int_{v_1}^{v_2} e^{-h_0^2v^2}dv, \quad . \quad . \quad . \quad . \quad (3)$$

subject to the condition

$$N_0+N_1+N_2=N,$$

is to be added to the expression (2), provided that this third drift has no systematic motion with reference to the stars in general.

In contrast to these three assumptions Schwarzschild has proposed an altogether different conception of stellar motion in his well-known ellipsoidal hypothesis. Applied to radial velocities, Schwarzschild's hypothesis requires that in any specified region the distribution of velocities should be represented by a *single* error function of the type (1). But in this case the modulus h should be a function of the position of the region in reference to the vertex or major axis of the ellipsoid.

A statistical test of these different hypotheses by observational data regarding radial velocities has now been rendered possible by a comprehensive collection of radial motions initiated by Mr. J. Voûte some years ago and recently completed. At my suggestion, and with the consent of H.M. Astronomer, Mr. Voûte undertook the formation of a catalogue of radial velocities of all the stars found in publications accessible to him at the Cape Observatory. He kindly placed the provisional results at my disposal for the purpose of the special investigation here indicated.

Mr. Voûte's catalogue comprises 1888 stars, for which he gives the approximate positions, visual magnitudes, spectral types, and observed radial velocities. The stars are arranged in separate groups according to type, showing the following total numbers:

Type.	Number of Stars.
B	312
A	342
F	251
G	308
K	535
M	140
Total,	1888

When collected in order of apparent magnitude the following distribution is obtained:

Stars Brighter than m	Number of Stars. B.	A.	FG.	KM.	Total.	Total Number of Stars (Kapteyn).
1	2	3	4	1	10	12
2	11	7	7	8	33	39
3	29	25	22	36	112	133
4	67	51	66	129	313	436
5	181	166	251	411	1009	1470
5·5	265	250	346	508	1369	2640
6·0	292	308	421	574	1595	4730
6·5	307	328	464	613	1712	8450

Compared with the last vertical column which represents the total number of stars in the sky according to Kapteyn, the table, as was to be expected, reveals a rapidly increasing decline in the number of stars with known radial velocities as the fainter magnitudes are approached. This of course is mainly due to the fact that our present knowledge of the radial velocities of faint stars is still very limited.

As we are concerned with the motions of the individual stars relatively to the centroid of the stellar system in its entirety, the radial components of the parallactic motions due to the transference of the solar system should be eliminated. If $V_\odot$ denotes the sun's velocity in km./sec. directed towards a point (apex) whose co-ordinates are A (right ascension) and D (declination), the radial component of this motion for a star whose co-ordinates are α, δ is expressed by:

$$v_\odot = -V_\odot[\sin\delta \sin D + \cos\delta \cos D \cos(\alpha - A)].$$

In accordance with Boss and Campbell we assume:

$$A = 270^\circ;\ D = +34^\circ;\ V_\odot = 20 \text{ km/sec.}$$

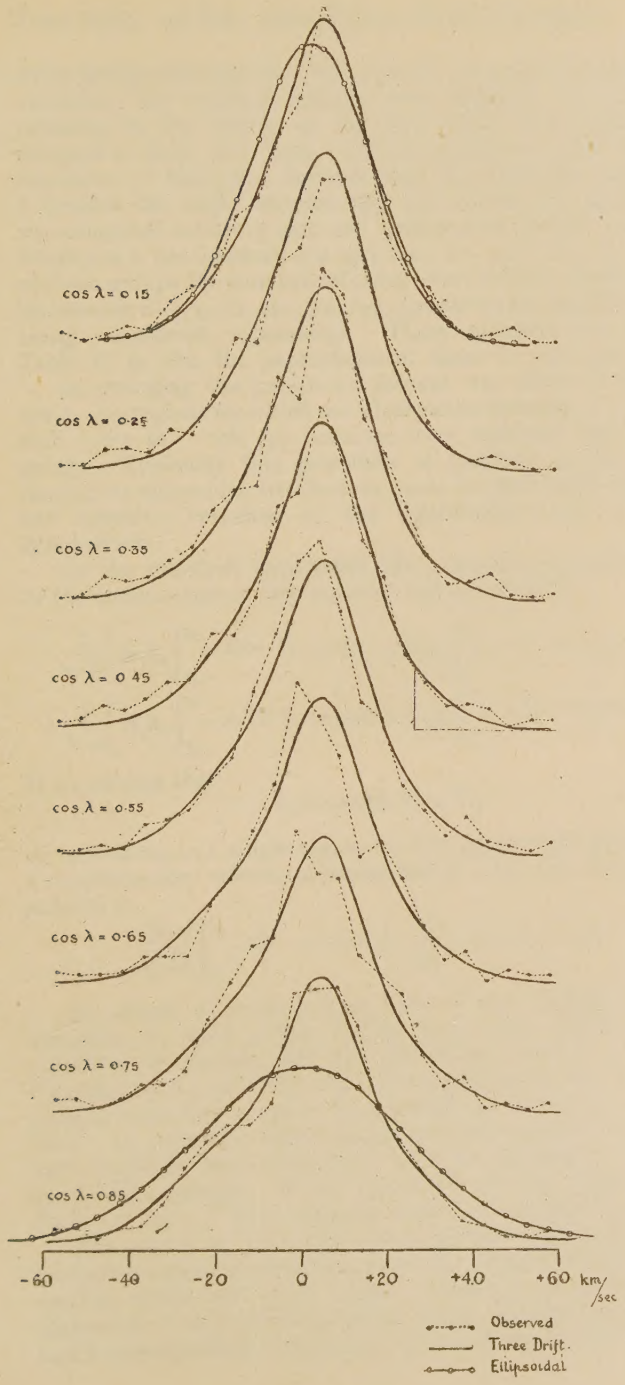

cos λ = 0·15
cos λ = 0·25
cos λ = 0·35
cos λ = 0·45
cos λ = 0·55
cos λ = 0·65
cos λ = 0·75
cos λ = 0·85
-60
-40
-20
0
+20
+40
+60 km/sec
Observed
Three Drift
Ellipsoidal

Fig. 1.

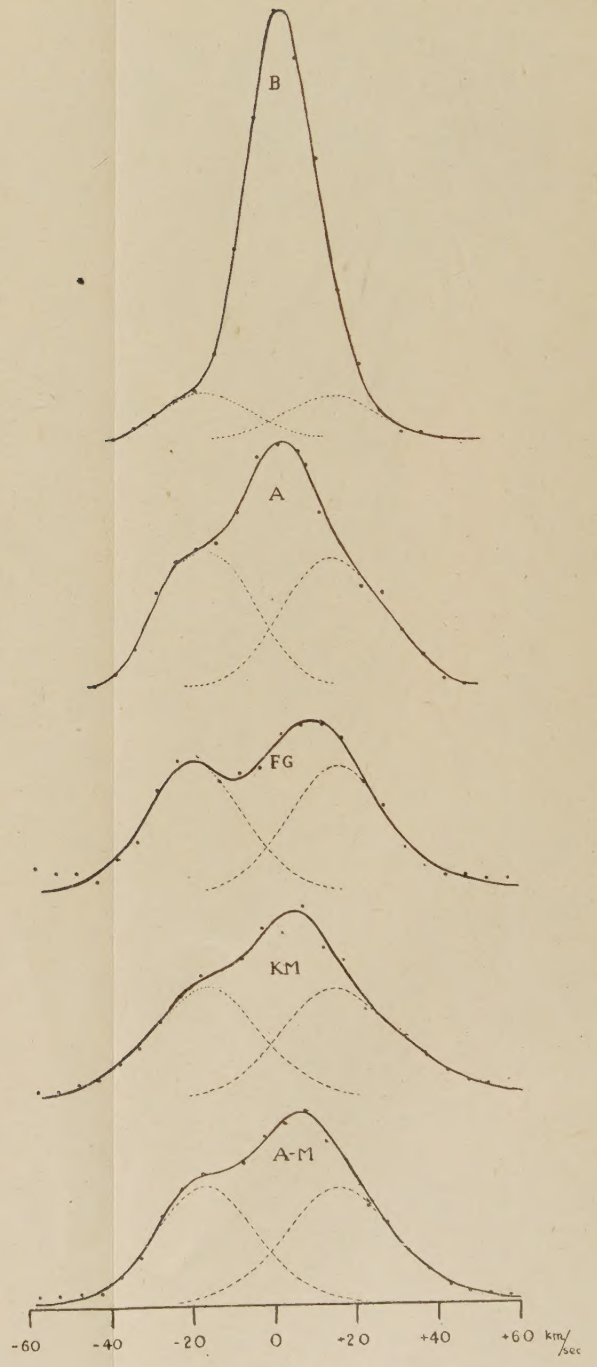
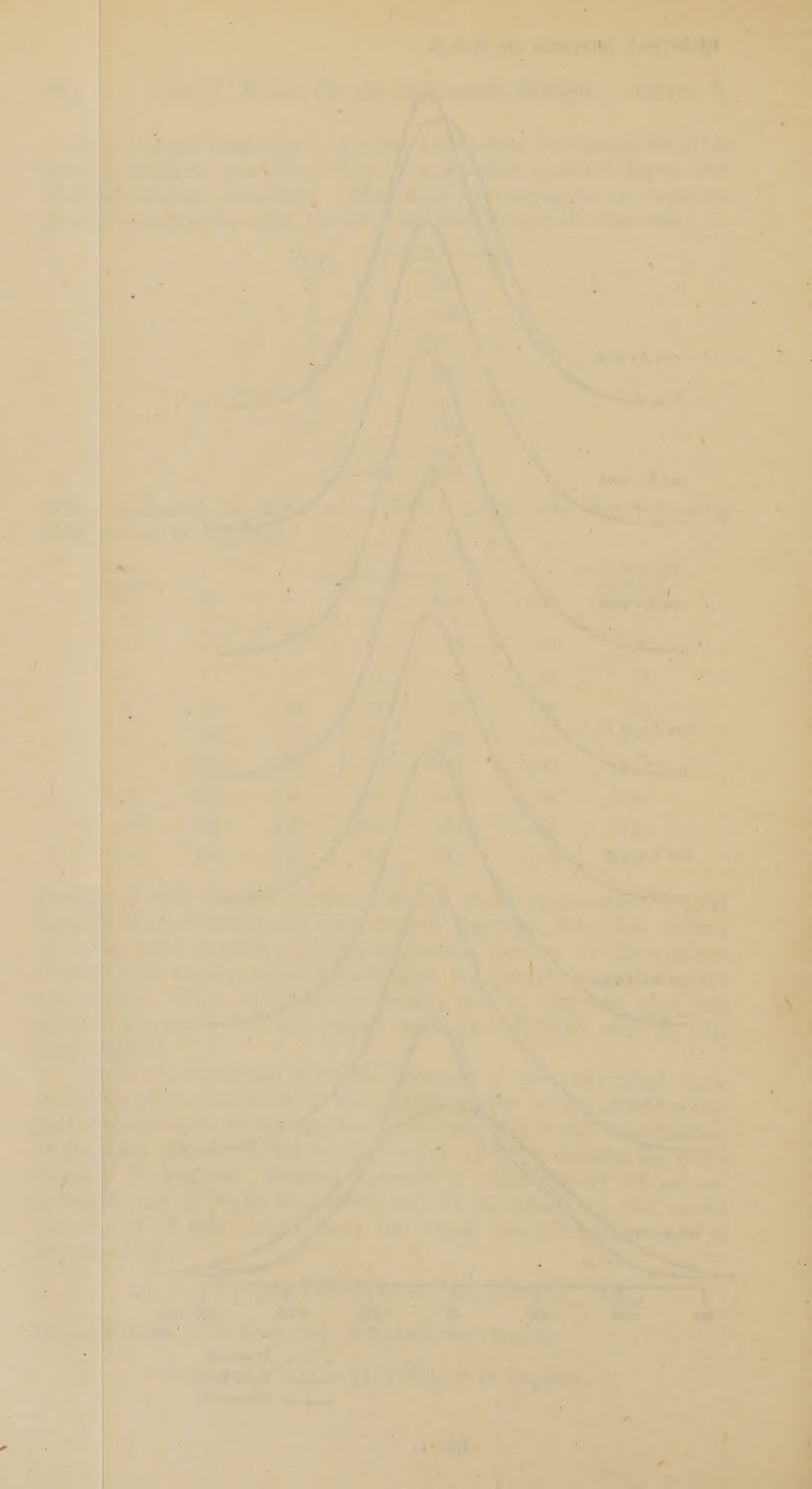

B
A
FG
KM
A-M
-60
-40
-20
0
+20
+40
+60 km/sec

Fig. 2.



After having deducted the values of $v_\odot$ from the individual radial velocities, the residual motions were arranged in groups with reference to the vertex of the two drifts whose position was assumed in R.A. 90°, Decl. +8°, in accordance with the determinations of this point derived from the transverse motions. If λ denotes the angle between star and vertex, the value of $\cos\lambda$ was computed for every star, and groups were formed of stars for which $\cos\lambda$ lies between 0·0 and 0·1; 0·1 and 0·2, etc. In each of these groups the numbers of stars were counted whose velocities lie between 0–5, 5–10, etc., km./sec. (positive and negative velocities being considered separately). These numbers are shown in Table I., as also the percentages of those in each group.

In arranging the groups no account was taken of the sign of $\cos\lambda$; *i.e.* regions for which $\cos\lambda$ had values between +0·5 to +0·6, and −0·5 to −0·6, for instance, were collected into the same group. Evidently this procedure is justified in the case of a random or ellipsoidal distribution, since in these cases the positive and negative branches of the distribution curve are strictly symmetrical.

In the two-drift hypothesis, the expression (2), when applied to the combination of two opposite regions, leads to:

$$\frac{1}{\sqrt{\pi}}\left[N_1 h_1 \int_{v_1}^{v_2} e^{-h_1^2(v-V_1\cos\lambda)^2}dv + N_2 h_2 \int_{v_1}^{v_2} e^{-h_2^2(v+V_2\cos\lambda)^2}dv\right]$$
$$+\frac{1}{\sqrt{\pi}}\left[N_1 h_1 \int_{v_1}^{v^2} e^{-h_1^2(v+V_1\cos\lambda)^2}dv + N_2 h_2 \int_{v_1}^{v^2} e^{-h_2^2(v-V_2\cos\lambda)^2}dv\right]. \quad (4)$$

If we assume that

$$h_1 = h_2 \text{ and } V_1 = V_2,$$

on combining in a single region the two regions obtained by giving λ supplementary values, supposing that $N = N_1 + N_2$, this expression reduces to

$$\frac{Nh}{\sqrt{\pi}}\left[\int_{v_1}^{v_2} e^{-h^2(v-V\cos\lambda)^2}dv + \int_{v_1}^{v_2} e^{-h^2(v+V\cos\lambda)^2}dv\right] \quad . \quad (5)$$

To obtain a more concise idea of the distribution of the velocities in each group, curves have been plotted with the velocities as abscissæ and the percentage numbers as ordinates. Considering the relatively small total number of stars in the single groups, these curves cannot be expected to show a smooth appearance. In order to reduce accidental discrepancies, *mean* curves have therefore been constructed from the material of every three consecutive groups. These are shown in the dotted curves of Plate 12, fig. 1. They comprise stars of *all* spectral types.

The curves on the whole reveal at first sight a much greater similarity among themselves than the results of the transverse motions would lead us to expect. There is, no doubt, a gradual depression of the maximum as the vertex ($\cos\lambda = 1$) is approached, and a corresponding broadening of the curves at their base. But

TABLE I.

Vel. Km./sec.		Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.	Nos.	Percentages.
>	−60	1	0·6	3	1·8	8	4·1	1	0·7	1	0·6	3	1·4	5	2·7	7	3·5	5	2·5	2	0·9
−60 to	−55	1	0·5	1	0·6	0	0·0	0	0·0	0	0·0	2	0·9	0	0·0	2	1·0	2	1·0	0	0·0
−55	−50	0	0·0	0	0·0	0	0·0	1	0·7	0	0·0	1	0·5	1	0·5	1	0·5	2	1·0	1	0·5
−50	−45	1	0·5	0	0·0	1	0·5	3	2·1	1	0·5	1	0·5	1	0·5	1	0·5	0	0·0	0	0·0
−45	−40	1	0·6	1	0·6	2	1·0	2	1·4	0	0·0	2	0·9	0	0·0	2	1·0	2	1·0	0	0·0
−40	−35	1	0·6	1	0·6	1	0·5	2	1·4	2	1·1	3	1·4	4	2·1	1	0·5	3	1·5	1	0·5
−35	−30	5	2·7	4	2·4	1	0·5	4	2·8	4	2·2	3	1·4	3	1·6	2	1·0	3	1·5	6	2·8
−30	−25	7	3·9	2	1·2	5	2·6	2	1·4	6	3·3	4	1·9	2	1·1	2	1·0	8	3·9	12	5·6
−25	−20	6	3·3	4	2·4	5	2·6	8	5·6	8	4·4	7	3·3	5	2·7	10	5·1	11	5·4	8	3·7
−20	−15	7	3·9	11	6·5	14	7·2	7	4·9	6	3·3	10	4·7	11	5·9	8	4·0	17	8·3	9	4·2
−15	−10	12	6·6	14	8·3	11	5·6	6	4·2	11	6·0	17	8·0	16	8·6	10	5·1	14	6·8	10	4·7
−10	− 5	20	11·0	16	9·5	19	9·8	14	9·8	21	11·5	21	9·9	18	9·7	17	8·6	13	6·3	10	4·7
− 5	0	22	12·1	24	14·2	16	8·2	13	9·1	20	11·0	28	13·1	32	17·2	24	12·1	21	10·2	27	12·6
0	+ 5	33	18·1	21	12·4	33	16·9	17	11·9	32	17·6	33	15·5	21	11·3	22	11·1	23	11·2	28	13·0
+ 5	+10	25	13·8	22	13·0	29	14·8	19	13·3	27	14·8	20	9·4	19	10·2	25	12·6	21	10·2	28	13·0
+10	+15	17	9·4	21	12·4	14	7·2	13	9·1	17	9·3	17	8·0	8	4·3	11	5·6	25	12·2	27	12·6
+15	+20	9	5·0	8	4·8	13	6·7	12	8·4	10	5·5	16	7·5	12	6·5	18	9·1	8	3·9	12	5·6
+20	+25	3	1·7	6	3·5	9	4·6	9	6·3	4	2·2	4	1·9	11	5·9	14	7·1	8	3·9	7	3·3
+25	+30	5	2·8	2	1·2	6	3·1	4	2·8	1	0·6	7	3·3	5	2·7	5	2·5	6	2·9	10	4·7
+30	+35	2	1·1	3	1·8	1	0·5	2	1·4	1	0·5	3	1·4	2	1·1	2	1·0	6	2·9	6	2·8
+35	+40	0	0·0	1	0·6	1	0·5	1	0·7	3	1·6	3	1·4	5	2·7	3	1·5	2	1·0	1	0·5
+40	+45	0	0·0	1	0·6	2	1·0	1	0·7	4	2·2	0	0·0	0	0·0	1	0·5	1	0·5	3	1·4
+45	+50	2	1·1	1	0·6	2	1·0	0	0·0	0	0·0	1	0·5	2	1·1	1	0·5	0	0·0	1	0·5
+50	+55	1	0·5	0	0·0	0	0·0	1	0·7	0	0·0	2	0·9	0	0·0	1	0·5	0	0·0	1	0·5
+55	+60	0	0·0	1	0·6	0	0·0	0	0·0	2	1·1	1	0·5	1	0·5	1	0·5	1	0·5	1	0·5
>	+60	1	0·6	1	0·6	2	1·0	1	0·7	1	0·6	4	1·9	2	1·1	7	3·5	3	1·5	4	1·9
		182		169		195		143		182		213		186		198		205		215	

the general impression is gained that a one-drift hypothesis would seem to be in better agreement with the observations than a two-drift assumption.

A closer analysis, however, shows that the former hypothesis cannot be accepted even as a rough approximation. It will be noticed, for instance, that in nearly all the curves the slope of the descending branch is distinctly steeper than that of the ascending branch. A feature of this kind is not accounted for by either the random or ellipsoidal distribution which demand curves symmetrical to the maximum ordinate.

To show, as an example, the conspicuous differences between the observed distribution and that computed from the ellipsoidal theory, the curves constructed from the latter for regions at the vertex (cos $\lambda = 1.0$) and at 90° from the vertex (cos $\lambda = 0$) are exhibited in the diagrams of fig. 1 corresponding with cos $\lambda = 0{\cdot}85$ and $0{\cdot}15$ respectively, the moduli of the Schwarzschild function being taken from Eddington, *Stellar Movements*, p. 134. The agreement with the observed curves, especially near the vertex, is doubtless very unsatisfactory.

The same criticism applies to the two-drift hypothesis. On the other hand, the assumption of *three* drifts leads to what may be considered a fairly satisfactory representation of the observed distributions.

In accordance with this hypothesis it is supposed that in each group the total number of stars whose velocities lie between v_1 and v_2 can be represented as the sum of three error integrals of the following form:

$$\frac{N}{\sqrt{\pi}}\Big[0{\cdot}35\int_{v_1}^{v_2} e^{-h^2(v-V\cos\lambda)^2}d(hv) + 0{\cdot}35\int_{v_1}^{v_2} e^{-h^2(v+V\cos\lambda)^2}d(hv) + 0{\cdot}25\int_{v_1}^{v_2} e^{-k^2(v-V_0)^2}d(kv)\Big] \quad . \quad (6)$$

where

$$h = 0{\cdot}046\,;\ V = 16{\cdot}4 \text{ km./sec.}\,;\ k = 0{\cdot}092\,;\ V_0 = 4{\cdot}0 \text{ km./sec.}$$

The calculated distributions are shown in the drawn-out curves of fig. 1 which generally represent the characteristic features of the observed curves as well as can be expected, considering the scanty statistical material on which the latter are based. Some of the curves show a slight displacement of the maximum which could be considerably reduced by appropriate changes in the value of V_0. Some systematic differences are also noticeable in the region of large velocities where the observed numbers as a rule exceed the computed values. About 5 per cent. of the total number of stars are thus not accounted for. The discrepancy may be explained by the assumption that each of the first two integrals in (6) should be replaced by a sum of two or more integrals of the same type with different moduli h. An assumption of this kind is justified if the drift is composed of stars of

different *masses* (giants and dwarfs) whose velocities are distributed in accordance with Maxwell's law. The material is, however, scarcely sufficiently reliable to carry the analysis beyond the stage in expression (6).

It will be noticed that the first two integrals of (6) indicate the presence of two drifts moving in opposite directions with the velocities $\pm V$ relatively to the centroid of the entire stellar system. The velocity of the one drift relatively to the other is therefore $2V = 32{\cdot}8$ km./sec. According to Eddington (*Stellar Movements*, p. 102) the transverse motions give for this velocity $1{\cdot}87/h$ and $1/h = 20$ km./sec.; hence $V = 37{\cdot}4$ km./sec., which is in fairly good agreement with the value here found.

Drift O is represented by the third integral of (6). The stars of this drift appear to possess a common motion of recession amounting to about 4 km./sec. in all parts of the sky. It is this systematic motion (if the spectral displacement from which it is inferred is indeed due to motion) which causes the asymmetric appearance of the curves, and gives rise to the K-term pointed out by Campbell. The "average" velocity $\bar{v}$ of the stars of Drift O can be found from the modulus k by the relation:

$$\bar{v} = \frac{\int_{-\infty}^{+\infty} e^{-k^2v^2} v\, dv}{\int_{-\infty}^{+\infty} e^{-k^2v^2} dv} = 1/k\sqrt{\pi} = 6{\cdot}1 \text{ km./sec.},$$

which agrees with the value found by Campbell for the stars of type B. The result supports the conclusion already deduced from the transverse motions that the B-type stars are closely associated with Drift O. But it was pointed out at the time (*M.N.*, 71) that the third drift is by no means exclusively confined to this particular type. Even after excluding the B-type stars, the transverse proper motions of the remaining stars of the Boss Catalogue show undoubted evidence of Drift O. It is therefore important to investigate the distribution curves of the radial velocities with special reference to type in order to examine to what extent Drift O makes itself felt in types other than B.

Naturally a differentiation between the drifts is best possible in regions near the vertex where Drifts I. and II. are most widely separated. For this reason the investigation has been confined to areas in which $\cos\lambda > \pm 0{\cdot}7$. The resulting distribution-curves for stars of the types B, A, F–G, KM and A–M are shown in Plate 12, fig. 2, where the ordinates of the dots (.) represent the observed percentages of stars between velocities 0–5, 5–10, etc., km./sec. (plus and minus).

A graphical analysis was effected in the following manner. The branches of the observed curves in the region of large velocities (positive as well as negative) are not sensibly affected by Drift O, and therefore represent the two-drift phenomena in its purest form. For this reason they were utilised for construct-

ing curves symmetrical to ordinates lying about 16 kms. on either side of the zero point of abscissæ, these curves being assumed to represent the distribution functions of Drifts I. and II. (The accepted displacements ± 16 kms. represent the two drift motions in the selected region relatively to the stellar centroid, viz., according to Eddington, $\frac{1\cdot87}{2h}\cos\lambda = 1\cdot87 \times 10 \times 0\cdot85 = 15\cdot9$.)

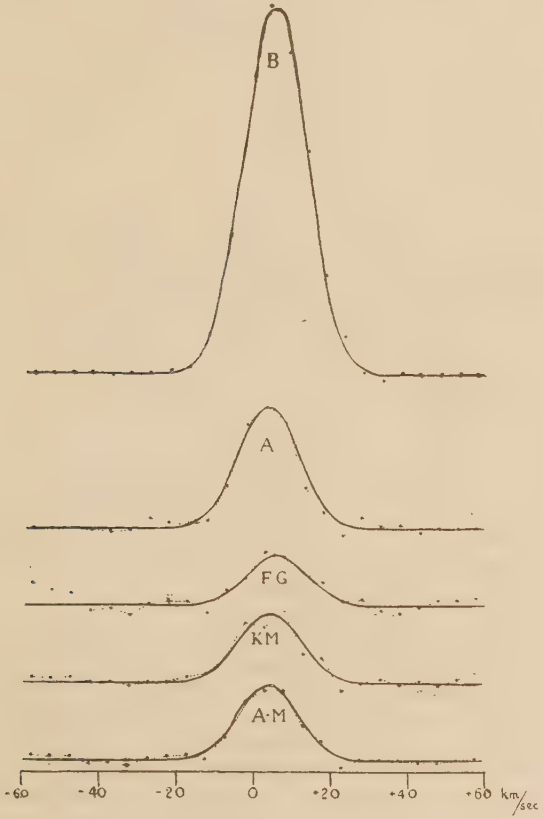


Fig. 3.

The curves constructed in this manner are shown in fig. 2 in the dotted curves.

It was found that the combination of the two curves could not be made to satisfy the *observed* curves in their central portions, pointing to the conclusion that the two-drift hypothesis is not sufficient

The differences between the observed ordinates and those obtained by adding the corresponding ordinates of the two drift curves are exhibited in the curves of fig. 3. They show con-

clusively that in all types there exists an additional drift whose intensity (amplitude) varies considerably from type to type. The curves can be well represented by simple error curves of the form:

$$\alpha \frac{k}{\sqrt{\pi}} \int_{v_1}^{v^2} e^{-k^2(v-V_0)^2} dv,$$

where $k = 0.046$, and V_0 and α have the following values:

Type	B	$V_0 = +4{\cdot}0$ km./sec.	$\alpha = 76$
	A	$+3{\cdot}0$	25
	FG	$+5{\cdot}5$	10
	KM	$+3{\cdot}8$	14
	AM	$+3{\cdot}5$	15

These error curves are shown in fig. 3 by the drawn-out curves which pass closely through the points of observation. The drift here indicated shows the characteristic features of Drift O found from the general curves of fig. 1, having the same modulus k and exhibiting a practically constant positive displacement of about 4 km./sec. Assuming the total number of stars to be represented by 100, the numbers α signify the total of stars in each type belonging to Drift O.

We may now conversely deduct from the observed star numbers in each type the numbers of Drift O, computed from the above error integral. The residual curves should satisfy the conditions of a pure two-drift phenomenon. How far this is the case is shown in fig. 4. It will be noticed at once that these residual curves cannot be satisfied by a single error curve such as the ellipsoidal theory would demand.

The special examination of types confirms the result derived from the preceding general discussion, viz. that a satisfactory representation of the observed distribution of radial velocities is obtained on the basis of the three-drift hypothesis advocated in *M.N.*, 71, whereas the two-drift and ellipsoidal hypotheses cannot be accepted even as rough approximations.

But there is one significant feature brought out by this special investigation which deserves particular consideration, viz. the considerable variations in the relative proportions of the drifts from one type to another. Drift O holds a most predominant position in the B-type, where the two-drift phenomenon is only feebly indicated. The opposite extreme is represented in types FG, and it is further significant that there appears a slight but distinct revival of Drift O in the more advanced types KM. This fact militates against the assumption that spectral evolution *per se* represents a discriminating feature in the relative mixture of the drifts.

A valuable clue, however, probably pointing to the correct explanation, is afforded by the mean parallaxes of the types.

According to Campbell these are (see Eddington, *Stellar Movements*, p. 167):

Type	B	$\pi = 0''.0076$
	A	0".0166
	FG	0".0302
	KM	0".0140

The correspondence between these parallaxes and the relative frequencies of the drifts is pronounced. The two-drift phenomenon

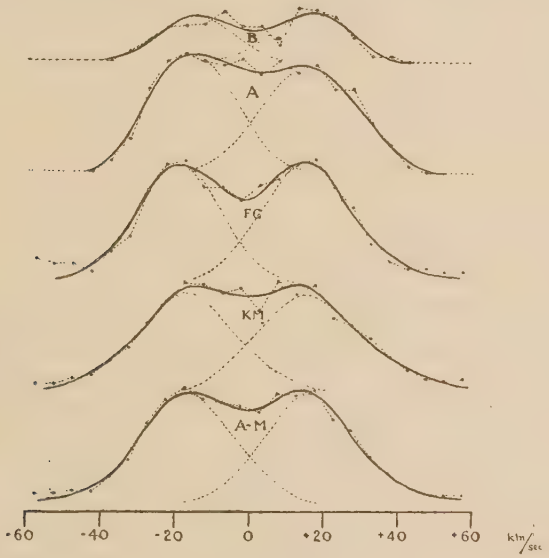


Fig. 4.

is most prominent among the nearest stars (types F and G) and least noticeable among the most distant stars (type B). Naturally the exactly opposite conclusion applies to Drift O.

This significant result is corroborated by similar evidence obtained from the transverse motions. The investigation of drift motion from the large transverse p.m. of Porter's Catalogue by Dyson (*Proc. Roy. Soc. Edin.*, 28) reveals no evidence of Drift O, whereas the generally more distant stars of the Boss Catalogue show its existence unmistakably. Dyson's results are supported by recent investigations of large p.m. by Innes and Wood confined to stars on selected Cape Astrographic plates, whose motions could be detected by the "blink" method. As doubtless the amount of angular p.m. considered on the average is a safe criterion for comparative nearness, the inference seems justified that near the

sun the motions of the stars are accounted for by the assumption of two drifts in accordance with Kapteyn's original hypothesis. The further we recede from the sun, however, the more will the presence of a third Drift O make itself felt, until, when the average distances of the B-type stars are reached, the observed motions are almost completely represented by this single drift.

The results derived from the present investigation may be thus summarised.

1. The observed distribution of the radial velocities cannot be accounted for by either the two-drift or the ellipsoidal hypothesis. The only assumption leading to a fairly satisfactory representation of the observations is a three-drift hypothesis in accordance with that derived in *M.N.*, 71, 610, from the tranverse motions.

2. The double-drift phenomenon (Drifts I. and II.) is of a distinctly *local* character chiefly concerning the stars in the vicinity of the solar system. Stars at considerable distances from the sun belong pre-eminently to Drift O.

Enhanced Lines of Nitrogen in the Region H_β to H_ϵ.

By A. Fowler, F.R.S. (Plate 13.)

The occurrence of enhanced lines in the spectrum of nitrogen was first noted by Lockyer, Baxandall, and Butler in an important paper published in 1909,* where it was shown that three such lines, at wave-lengths 4097·4, 4103·5, and 4379·8, were present in the spectra of stars of early B type, in contrast with the appearance of unenhanced lines of the same element in the spectra of B stars of later type. Further attention to these lines has recently been drawn by Dr. J. Lunt in his discussion of the emission bands of Nova Aquilae No. 3,† from which it appears highly probable that enhanced nitrogen lines were present in the nova at a stage following that in which enhanced metallic lines were predominant, and were possibly also accompanied by some of the unenhanced lines. Particular attention was drawn by Lunt to the desirability of further experimental work in connection with the interpretation of the well-known nova band about λ 4640.

A general investigation of the line spectra of nitrogen and other gases has been in progress at the Imperial College for some years, more particularly with a view to associating the changes in the spectrum with changes in type of series, but the work in the far ultra-violet is still incomplete. Pending further experimental work, however, Dr. Lunt's paper has suggested that it may be useful to give the results for nitrogen which are directly applicable to the interpretation of celestial spectra in the region most generally photographed.‡

* *Proc. Roy. Soc.*, A, **82**, 532. † *M.N.*, **80**, 534 (1920).

‡ Some of the results for oxygen have already been published. A. Fowler and J. Brooksbank, *M.N.*, **77**, 511 (1917).

The ordinary line spectrum is readily obtained by passing discharges between metallic electrodes in air or nitrogen at atmospheric pressure, or by condensed discharges of moderate intensity through nitrogen at a pressure of a few millimetres in vacuum tubes. The enhanced lines become prominent when strongly condensed discharges are employed, and their appearance, as in the case of the enhanced line of helium 4686, and the third line spectrum of oxygen, is facilitated by low pressure of the gas in the tubes. Many of the nitrogen lines which are broad and diffuse at the higher pressures become sharply defined and capable of precise measurement when the pressure is suitably reduced. Numerous photographs have been obtained over a considerable range of electrical conditions, and with instruments of various degrees of dispersion, including the first and third orders of a 10-ft. concave grating.

Several enhanced lines of nitrogen have been observed in addition to those mentioned by Lockyer, Baxandall, and Butler, and the wave-lengths of all the stronger lines in the region extending from H_β to H_ϵ are collected in the third column of Table I. The opportunity has also been taken of revising and tabulating the wave-lengths of the chief unenhanced lines which occur in the same region. Many of the lines were previously included in a list given by J. S. Clark from determinations of air lines in the third order of the concave grating;* these have been substantially confirmed by further measures, and Clark's wave-lengths have been adopted. When possible, the same excellent plates have been used for the additional lines. Lines on the less refrangible side of 4643 have been determined from a first order photograph, and the remaining wave-lengths have been derived either from this or from plates taken by Mr. J. Brooksbank with a large Littrow prismatic spectrograph giving a dispersion ranging from 4 A per mm. at λ 4600 to 1·9 A per mm. at λ 4000. The spectra on the Littrow plates are practically free from impurities of oxygen and carbon (see Plate 13).

The wave-lengths were determined in the first instance with reference to international standards, but for the convenience of astronomers they have been reduced to the Rowland scale by the use of Kayser's corrections. It is believed that the wave-lengths will rarely be found to be more than 0·01 A or 0·02 A in error.

The intensities have been estimated throughout from the Littrow plates. Nearly all the lines tend to be diffuse, except at the lowest pressures, and only the most nebulous lines are indicated by the letter *n* following the intensity.

Although the enhanced lines have not been experimentally produced without admixture with the ordinary lines, analogy with enhanced lines of other elements strongly indicates that they may exist alone in stellar spectra when the conditions are favourable. Thus, in stellar spectra the enhanced lines may in some cases appear in the absence of unenhanced lines, and *vice versa*, or the

* *Astrophys. Jour.*, 40, 332 (1914).

TABLE I.

The Brighter Lines of Nitrogen in the Region $H\beta$ *to* $H\epsilon$.

(Wave-lengths on Rowland's Scale.)

Ordinary Lines.		Enhanced Lines.		Remarks.
Wave-length.	Intensity.	Wave-length.	Intensity.	
		4867·32	4	*
		4859·00	3	
4803·48	6			
4788·31	5			
4779·91	4			
4643·28	7			† (=λ Clark)
		4642·08	3	
		4640·82	10	†
		4634·34	8	†
4630·73	10			†
4621·58	7			†
4614·06	6			†
4607·34	7			†
4601·66	8			†
4552·71	4*n*			
		4546·50	2	
		4535·24	1	
		4534·74	2	
4530·59	6*n*			†
		4523·76	3	
		4518·32	2	
		4515·04	6	†
		4511·08	5	†
4507·75	3			
4447·20	10*n*			†
4432·93	6*n*			Fainter companion on red side.
		4379·26	10*n*	A faint ordinary line at 4379·85.
		4346·01	1	
4241·94	5*n*			†
4237·15	4*n*			†
4227·90	2*n*			†
		4200·22	4	
		4195·82	2	
4179·84	1*n*			
4176·31	2*n*			†
4171·78	1*n*			
		4103·54	8	†
		4097·48	10	†
4043·69	2*n*			
4041·47	6*n*			†
4035·24	4*n*			†
4026·23	2*n*			†
		4003·60	2*n*	
		3998·80	1*n*	
3995·15	10			†

* There is another line at 4861·50, sensibly coincident with $H\beta$, which does not seem to be certainly accounted for by a possible impurity of hydrogen.

two sets of lines may co-exist with different proportionate intensities. When both sets occur together, it may be assumed, provisionally, that the intensities of the lines of each set will change as a whole, so that the relative intensities in either set will remain unchanged.

While detailed discussion of the nitrogen lines in relation to the spectra of Novæ can only be properly entered into by those who have access to original negatives, a few remarks on this subject may perhaps be permitted. The evidence produced by Lunt for the presence in Nova Aquilæ No. 3 of the enhanced lines 4097 and 4379 is strengthened by a consideration of the details of the "4640" band of the nova. Lunt's careful analysis of the nova spectrum has led him to conclude that the very broad band extending from 4578 to 4666 consists of at least five components, the centres of which are at 4604, 4634, 4641, 4650, and 4686. It now appears that 4634 and 4641 may be almost certainly identified with the strong enhanced nitrogen lines 4634·34 and 4640·82, which were not mentioned by Lockyer, Baxandall, and Butler. The band 4650 is doubtless to be accounted for by the pair of specially enhanced carbon lines at 4650·8, 4647·6 observed by Lockyer, Baxandall, and Butler, while 4686 corresponds with the enhanced line of helium 4685·98. The origin of the remaining component of the group, 4604, remains unknown.

Lunt's suggestion that the nova band for which he gives the wave-length 4378·9 is due to enhanced nitrogen is also strengthened by the new determination of the position of this line in the laboratory spectrum. Lockyer, Baxandall, and Butler apparently assumed that the enhanced line was identical with the faint line tabulated by Exner and Haschek at 4379·75, but this is not the case. The true wave-length of the enhanced line is 4379·26, while the other is an ordinary line, at wave-length 4379·85; the two lines are clearly separated in the spectra photographed with the grating, for which the pressure was very low. The new wave-length has also been confirmed by measures of the middle of the broad strong line in the plates taken with the Littrow spectrograph.

Further experiments may possibly indicate variations in the relative intensities of the enhanced lines given in the table, but meanwhile it is to be expected that all the stronger enhanced lines will appear together, if at all, in the nova spectrum. The evidence given by Lunt for the presence of the line 4097 appears to be sufficiently definite, and the apparent absence of its companion line 4103 is satisfactorily accounted for by its superposition on the broad H_δ band. Of the remaining lines in the region H_β to H_ϵ the most important are the pair at 4515·04(6) and 4511·08(5). Lunt does not mention a band in this part of the spectrum, but it would be less intense than the others, and it is not clear that his discussion includes all the bands which were present.

The additional evidence now presented, however, strongly supports Lunt's conclusion that enhanced lines of nitrogen were

represented among the bright bands which appeared in the spectrum of Nova Aquilæ in its later stages, and possibly also account for some of the bands which occur in the spectra of Wolf-Rayet stars.

[*Note, added July 8.*—Mr. F. E. Baxandall has informed the author that the abnormality of the nitrogen lines 4514·95, 4634·20, 4640·70, was recognised by him some years ago. Early in the present year Mr. Baxandall had noted that Wright's recent list of nebular lines (*Pub. Lick Obs.*, 13) included two lines at 4634·3, 4641·1, and had informed Mr. Wright of their probable identity with two of the enhanced lines of nitrogen, and with two lines recorded by Paddock in the spectrum of the Wolf-Rayet star A.G.C. 22748 (*Lick Obs. Bull.*, 8, 78).]

Description of Plate.

The photographs are reproduced from plates taken with the large Littrow spectrograph, and are on a scale 1½ times that of the original negatives. Iron arc comparison spectra are impressed above and below the nitrogen spectra throughout, one exposure being made before, and the other after, that of the gas.

In each section, the upper gas spectrum is that obtained with a strongly condensed discharge and low pressure, and the lower one that given by a condensed discharge of moderate intensity through the gas at a higher pressure.

The enhanced lines are those which are relatively stronger in the upper than in the lower spectrum of the gas.

The Spectra of Nova Aquilæ No. 3. (*Third Paper.*)

By Joseph Lunt, D.Sc. (Plate 14.)

(*Communicated by S. S. Hough, F.R.S., H.M. Astronomer at the Cape.*)

The present paper deals mainly with the changes which had taken place in the Nova spectrum when the star again came into view in 1919, and more particularly with the structure of the bright nebular band λ4363, which is shown to have the same structure as the chief nebular bands N_1 and N_2 observed by Moore and Shane * at the Lick Observatory, and other bands observed by Adams and Joy † at Mount Wilson.

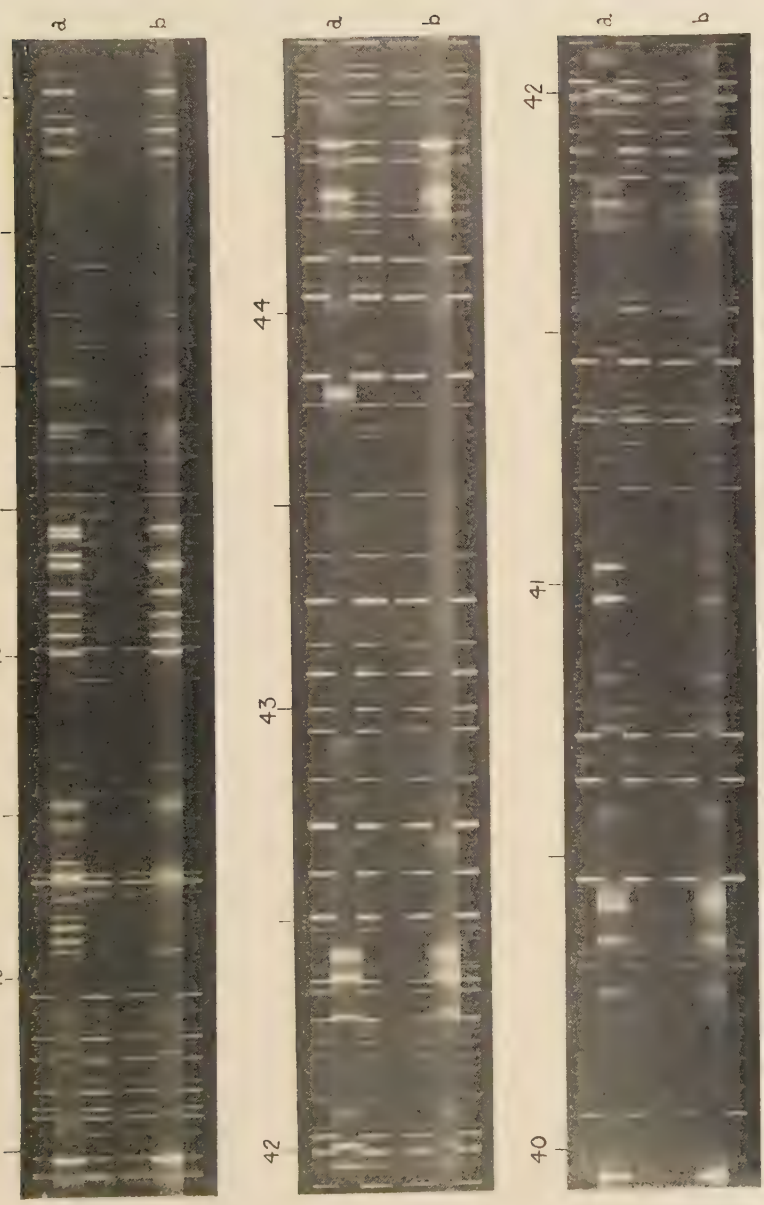
It also deals with the extraordinary pair of strong dark lines at λλ4054, 4060 seen in the 1918 spectra, the fine H and K lines of Calcium, and the bright Helium band λ4471·6 and its associated dark lines.

A remarkable feature of the entire series of spectra is the extraordinary persistence of a dark line displaced about 450 km./s. to the violet, which can be traced in various bands from the end of 1918 June to the beginning of 1919 September.

There seems to be a growing tendency to regard the high

* *L.O.B.*, 10, 34. † *Pub. Ast. Soc. Pac.*, 31, 183.

ENHANCED LINES OF NITROGEN.



(a) Strong Discharge.

(b) Weaker Discharge.

velocities shown by the true absorption lines as real, and due to gaseous matter explosively ejected from the Nova in its earlier stages. This idea, enunciated by Adams * and by Evershed †, accords better with the facts than my suggestion of a very high-velocity star from outside our system entering a nebula on the outskirts of the galaxy.

Barnard's measures ‡ show that the Nova is developing a sensible disc like a planetary nebula, and it is therefore of interest to compare it with the planetary nebulæ, for which Van Maanen has given distances and absolute dimensions.§

Assuming the displacements of the edges of the bands to be a measure of the rate of expansion of the Nova into a planetary nebula, we have a velocity of radial expansion approximately the two-hundredth that of light (1500 km./s.).

The largest of Van Maanen's nebulæ is the Ring nebula in Lyra with an angular diameter of 80″, which would be crossed by light in 0·16 years.

The Nova would therefore take sixteen years to expand to the absolute dimensions of the Ring nebula if the above rate of expansion is maintained uniformly at the above figure. If at the same distance as the Ring nebula it would expand in diameter at the rate of 5″ a year.

Barnard's measures indicate an expansion of nearly 2″ in the first six months, but Wright's measures || from spectra show no further definite change during the following seven months, whilst Aitken ¶ in 1919 September places the diameter at 2″·38.

The smallest of Van Maanen's planetary nebulæ, NGC 7662, would be equalled in actual size after two years of such expansion. Further measures of the diameter of the disc of the Nova will be looked forward to with interest.

Van Maanen's latest ** absolute parallax of the Nova places it at about one-third the distance of the Ring nebula, where, on the above assumption, it should expand in diameter at the rate of 15″ a year, which is not in accord with the observations.

The Structure of the Bright Bands in 1919.

The Nova spectrum was photographed on four occasions during 1919, viz. :

			m		h m
Plate 5443	on April 3, 4, and 5	Exps.	170, 180, 180	Total	8 50
,, 5463	,, June 9, 10, and 12	,,	180, 180, 154	,,	8 34
,, 5484	,, July 22, 23, 24, and 25	,,	180, 180, 112, 120	,,	9 52
,, 5499	,, Aug. 27, 28, 30, and 32	,,	140,(50††),175, 145	,,	8 30

* *Journal Roy. Ast. Soc. Canada*, **13**, 75.
† *M.N.*, **79**, 482. ‡ *Ap. J.*, **49**, 199.
§ *Mt. Wilson Contributions to the National Acady. of Sciences No.* 56.
|| *L.O.B.*, **10**, 30. ¶ *Pub. Ast. Soc. Pac.*, **32**, 63.
** *Ibid.*, **31**, 284. †† Estimated effective exposure on a cloudy night.

The iron spark spectrum was impressed on the plate at the beginning and end of each exposure.

As seen in the 18-inch visual telescope the colour of the Nova at once arrested attention, being quite unlike any other star, as the colour was a vivid green, and when examined by a direct vision spectroscope the nebular bands N_1 and N_2 were seen to be the predominant feature of the spectrum on a much fainter continuous background.*

The photographs show the nebular band 4363 to be as intense as the chief nebular bands N_1 and N_2, but the hydrogen bands have almost vanished and the 4641 group has become but little stronger than the continuous spectrum.

Adams and Joy,† observing in February and March, make the important observation that the maxima and minima of the bands have the same velocity displacements in the different bands, or as they say the displacements are proportional to wave-length, and they give measures to show that the bands H_β, H_γ, N_1, N_2, 4363, 4640 and 4686 have the same structure. I think this is the first time that this observation has been made, and it is of great importance as showing that in this case the apparent absorption lines are not real, but may be interpreted as the effects of jets or streams of gaseous matter radiating from the Nova in various directions. The radial velocity components of these streams range from -1913 to $+1879$ km./s.

Adams and Joy's observations are borne out by the measures made on the band 4363 on the Cape photographs as compared with the measures of Moore and Shane‡ made on the bands N_1 and N_2 for which their instrument (The Lick 36-inch) was especially focussed.

As these latter bands were very badly placed as regards focus in the Cape instrument it was superfluous to measure them in view of Moore and Shane's results, but the band 4363 being in the region usually employed for radial velocity work here, measures of this band were made, and they supplement and confirm the Lick results in a remarkable manner.

A striking fact about these bands is the long period over which the structure is practically constant in its main features. The structure of the bands may be interpreted as a series of narrow bright bands with darker interspaces. Moore and Shane's measures are given as the wave-length of the bright maxima extending from 4929·9 to 5038·6 with only two dark lines. If these are analysed they are seen to be 9 pairs of bright bands and 1 dark pair with the same velocity displacements for each member of the pair due to N_1 and N_2 respectively, and 4 bright bands due to N_1 alone. The displacements range from -1766 to $+1898$. See Table I.

Adams and Joy give their results for each band as displacements in Ångstrom units.

* See Note in *The Observatory*, 42, 248.
† *Pub. Ast. Soc. Pac.*, 31, 183.
‡ *L.O.B.*, 10, 34.

In Table II. the results of the various measures are given as velocity displacements so as to be comparable with the Cape results and with each other. Figures in brackets are calculated minima from the means of two neighbouring maxima or calculated maxima from the means of two neighbouring minima.

The maxima or bright bands are in heavy type, and the minima or dark lines in ordinary type.

The Mount Wilson figures are the means from H_γ, 4363, H_β, N_2 and N_1. The Lick figures are the means from N_2 and N_1.

Some of the features of the structure of the bands seen from February to September 1919 may be traced back to early August 1918, and are seen to have persisted almost without change for over a year. This is particularly the case with the dark line or space measured by the Lick observers at −460 km./s. in N_1 and N_2. This dark line is prominent in H_γ and the nebular band 4363. In H_γ and the helium band 4471·6 it may be traced back to the end of 1918 June. It is occasionally measurable in H_δ and on July 2 in H_ϵ also.

Table III. gives the measures, and it is remarkable how the observations made at Ottawa by Harper * fill the gaps in the Cape Observations.

Harper's measures as they stand agree remarkably well with the Cape measures, but if the correction +20 km./s. has been applied for the velocity of approach of the Nova, as indicated on pp. 508, 509 of his paper, his figures will require a correction of −20 km./s. to make them comparable with the Cape figures.

Uniformity of procedure in this regard is desirable.

The Pair of Strong Dark Lines at λλ4054, 4060 (1918, *July* 15).

The above pair of absorption lines has aroused great interest owing to its extraordinary behaviour. Its sudden vanishings and reappearances, changes from dark to bright and sudden variations of wave-length have rendered it conspicuous. The available information concerning this pair has been collected in Table IV. Wave-lengths (omitting the figures 40 in each case) which are connected by a hyphen denote a band with the given limits. Wave-lengths connected by "&" denote two separate lines. Figures in heavy type denote the mean position of the band or pair of lines.

In the first period, 1918 June 15 to June 25, the pair of lines begins as a faint shading, increasing in strength to a broad dark band with very diffuse edges.

Differences in exposure and development and the diffuseness of the edges of the band probably account, to some extent, for the differences in wave-lengths shown, but there is a real variation, and later measures, when the pair of lines is shown more definitely, confirm this reality. There is a general trend to the violet in this

* *Journ. Roy. Ast. Soc. Canada*, **12**, 498.

period, and the measures by the four observers (extended in longitude from 5^hE. to 8^hW.) agree fairly well.

In the second period, June 26–29, Paddock * records the region as bright with a band at 4055. The Cape plates show a general brightening about this place, very broad and with indefinite boundaries, but Paddock's measures 4062–4075 appear to refer to the dark space between this region and the beginning of the extension of H_δ towards the violet, which latter in a former paper I attributed to the violet edge of the broad nitrogen band 4097.

In the third period, from July 1–8, the pair of dark lines becomes more definite, especially on July 2–7, and the lines show very pronounced variations in wave-length which are undoubtedly real and readily detectable without measurement (see Plate).

They are well shown in Plaskett's † plate of July 6.

In the fourth period, July 9–13, the pair again vanishes and the region becomes bright. Harper records a new band 4046–4053 and again records 4062–4075. These may be merely darker regions on each side of the brightened region of the real bands, and the region of the real bands may be in a critical state and varying from slight brightening to slight darkening.

In the fifth period, July 14–17, the dark pair attains its greatest intensity and sharpness, and a comparison of the measures again indicates real shifts which must have been very rapid if we can rely on Harper's measures 4050 and 4054 on July 15, whilst on the same day the Cape photograph shows the very intense pair at 4054 and 4060 (G.M.T. $9^h\ 37^m$). After July 17 up to August 11 the pair of dark lines vanishes and the region is bright again, with the centre of a bright band, according to Paddock, at 4054 to 4057. In this period Harper's wave-lengths probably have reference to the dark spaces on the two sides of the bright band. The Cape photographs lend no support to the idea that the pair of lines ever extend so far to the red as Harper's wave-lengths indicate, and Paddock's interpretation of the spectra as showing a bright band with centre at 4054 to 4057 seems to me more probably correct.

Baxendall considers that the pair of dark lines are the abnormal nitrogen lines 4097·4, 4103·4. He says "Paddock's pair of lines at the measured positions near 4061, 4068 were found, whenever the presence of other nitrogen lines in the spectrum made comparison possible, to be ascribable to the two abnormal nitrogen lines λλ4097, 4103 partaking of the same velocity displacements as many other well-authenticated nitrogen and oxygen lines in the spectrum of Nova Aquilæ."

I find it very difficult to accept this conclusion without further evidence.

When the pair of lines is best seen on the Cape photographs (July 15) we have wave-lengths 4054 and 4060 which, if due to the nitrogen pair, must be displaced in the mean −3167 km./s., a displacement shared by no other dark lines in the Cape photographs.

* *Pub. Ast. Soc. Pac.*, **31**, 56.

† *Journ. Roy. Ast. Soc. Canada*, **12**, 350.

On July 2 the wave-lengths are 4051 and 4056, corresponding to a still larger mean displacement of −3432 km./s. On these plates there are no indications of the characteristic strong triplet 4070–72–76 or other strong lines of oxygen with the same displacements as the dark pair, and, indeed, on no other of the Cape plates have oxygen lines been seen with this or any other displacement. Moreover, the strong abnormal nitrogen line 4379·8 which accompanies 4097, 4103 in laboratory spectra, and which if correspondingly displaced should fall on the bright H_γ band, does not accompany the very strong pair in the Nova spectra.

It would be of general interest, therefore, if Baxendall would publish reproductions and measures of his spectra to throw further light on this matter.

With the pair of lines λλ4054, 4060 (July 15) Paddock associates another pair at λλ4560, 4576. This latter pair can only be measured with tolerable accuracy on July 15, when it is much more diffuse, and much fainter than the first pair. The measured wave-lengths are 4559·8 and 4573·4.

The Helium Band λ4471·6.

This band is never prominent, but it is important to note its presence. From 1918 June 29 to July 6 it has a dark line on its violet edge which has the same displacement as the dark H_γ.

The following measures were made :—

Date 1918.	Plate No.	Dark Line. km./s.	Bright Band Begins.	Compared with	
June 29	5284	−1685	−1631	−1642	H_γ bright
July 1	5286	−1655	...	...	
2	5287	−1674	...	...	
2	5288	−1657	...	...	
6	5291	−1678	...	...	
10	5298	...	−1673	−1654	H_γ ,,
Sept. 12	5346	...	−1699	−1713	H_δ ,,
	Means	−1670	−1668	−1670	

June 28 to July 6. Dark H_γ −1672 (previous paper).

The red end of the band is definite enough to measure only on July 10 when it is displaced +1582. On the same date H_δ ends at +1583 and H_γ at +1544. The band lies therefore roughly between the iron lines λλ 4448 and 4495, and except on the above plates it is not sufficiently differentiated from the continuous spectrum for measurement.

As shown in Table III., the strong absorption line noted by the Lick observers in the N_1 and N_2 bands is present also in this Helium band. The band is absent in the 1919 photographs.

The Narrow H and K Lines.

As in previous Novæ the fine dark lines of Calcium close to their normal positions are a feature of the spectrum of especial interest. Some observers regard the displacement of these lines as a measure of the radial velocity of the Nova itself, and apply a correction for it to their measures of the greatly displaced lines. This view is probably an incorrect one, and, as Evershed * shows, the displacement both in Novæ and in other stars seems more likely to be due to the sun's motion with respect to very tenuous Calcium vapour pervading inter-stellar spaces, which is only revealed to us when the H and K lines due to the Nova or star are sufficiently displaced. Plaskett † expresses similar views. The measures are as follows, the second set being the means of four settings in the direct and reversed positions of the plate.

Date. 1918.	Plate No.	Lines.	Single Setting.	Four Settings.	Weighted Mean.
June 10	5260	K and H	−13·8	−17·0	−16·4
11	5266	,,	−14·6	−13·5	−13·7
12	5269	,,	− 9·1	−11·8	−11·3
15	5272	,,	−12·4	−14·2	−13·8
16	5273	H	−21·2	−24·9	−24·2
17	5275	K and H	−11·2	−15·4	−14·6
18	5276	H	−24·3	−25·8	−25·5
July 5	5289	,,	−15·7	−12·4	−13·1
5	5290	,,	−14·8	−23·1	−21·4
6	5291	,,	− 9·0	−15·1	−13·9
Weighted mean (K and H Wt 2, H Wt 1) . . .					−15·8±0·75
Correction for solar motion					+17·1
Radial velocity					+ 1·3

The above results show variations similar to those observed by Plaskett, but he does not consider real variation as established, and as in the Cape plates the region of H and K is outside that of good focus, the results may be taken as indicating a constant velocity which accords well with that obtained elsewhere as shown below.

	Plates.	Km./s.
Adams and Joy (*Pubns. Ast. Soc. Pac.*, **30**, 307)	14‡	−15
Harper (*J.R.A.S.C.*, **12**, 498)	19	−19·9±0·7
Plaskett (,, **12**, 353)	17	−19·7±0·4
Evershed (*M.N.*, **79**, 477)	(days) 4	−14
Lunt (this paper)	10	−15·8±0·75

* *The Observatory*, **42**, 86, and *M.N.*, **79**, 477.
† *Jour. Ast. Soc. Canada*, **12**, 354.
‡ Including D_1 and D_2.

Weighting in accordance with the number of plates the result is $-17\cdot8$, which is in good agreement with the component of the solar motion towards the Nova, viz. $-17\cdot1$ km./s.

Royal Observatory,
Cape of Good Hope,
1920 *April* 29.

TABLE I.

Moore and Shane's wave-lengths, N_1 and N_2 bands, Reduced to displacements.

$N_2 = \lambda 4959\cdot1$ $N_1 = \lambda 5006\cdot9$.

Measured λ.	Shift in λ.	Shift in Km./s.	Maxima Mean.	Minima (means of maxima).
N_2 4929·9	−29·2	−1766	−1755	
N_1 4977·8	−29·1	−1743		−1687
N_2 4932·4	−26·7	−1615	−1619	
N_1 4979·8	−27·1	−1623		−1443
N_2 4938·5	−20·6	−1246	−1267	
N_1 4985·4	−21·5	−1288		−1093
N_2 4944·3	−14·8	− 895	− 918	
N_1 4991·2	−15·7	− 940		
N_2 4948·9	−10·2	− 617	− 620	
N_1 4996·5	−10·4	− 623		
N_2 4951·4	− 7·7	− 466	(dark lines)	− 460
N_1 4999·3	− 7·6	− 455		(measured)
N_2 4954·3	− 4·8	− 290	− 289	
N_1 5002·1	− 4·8	− 287		
N_2 4959·3	+ 0·2	+ 12	+ 9	
N_1 5007·0	+ 0·1	+ 6		+ 136
N_2 4963·2	+ 4·1	+ 248	+ 262	
N_1 5011·5	+ 4·6	+ 275		
N_2 4970·8	+11·7	+ 707	+ 707	+ 485
N_1 5018·7	+11·8	+ 707		+ 890
N_1 5024·8	+17·9	+1072	+1072	+1246
N_1 5030·6	+23·7	+1419	+1419	+1509
N_1 5033·6	+26·7	+1599	+1599	+1749
N_1 5038·6	+31·7	+1898	+1898	

Note.—With one exception the photographs were taken with a one-prism spectrograph giving a linear dispersion at λ5000 of 68A to the millimetre.

The Cape plates were taken with a four-prism spectrograph giving a dispersion * at the 4363 band about three and a half times that of the Lick Instrument at λ5000.

* See *M.N.*, 79, 418.

TABLE II.

Structure of Bright Bands in 1919.

	Cape. Lunt. Nebular Band λ4363·4.					Mt. Wilson,* Adams & Joy, Hβ, Hγ, N_1, N_2 and 4363.	Lick,† Moore and Shane, N_1 and N_2 (See Table I.)
	April 3, 4, 5. Plate 5443.	June 9, 10, 12. 5463.	July 22–25. 5484.	Aug. 27–Sept. 1. 5499.	Mean.	Feb. and Mar. Mean.	July 28–Aug. 2.
	−1925*f*	−1913*f*	...	−1900*f*	−1913*f*	...	..
	1870	1851	...	1844	1855	...	...
	1725*s*	**1748***s*	**−1747***s*	**1756***s*	**1744***s*	−1720	−1755
[Hγ	**1748***f*	**1740***f*	...	...	1744]	...	...
	1629	1645	...	1647	1640	...	(1687)
	1533	1570	...	1581	1561	...	1619
	1464	1453	...	1469	1462	1446	(1443)
	(1330)	(1323)	1293	1301	1312	(1308)	1267
	1196	1192	1183	1183	1189	1170 }	(1093)
	..	...	1025	...	1025	... }	
	...	...	...	...	(936)	...	918
	853?	862?	826	...	847	...	...
	(640)	649	(617)	...	635	636	620
	427	408	407	438	420	466	460dark
	...	− 323	...	− 313	− 318	− 331	− 289
	...	...	...	...	...	39	+ 9
	+ 123	...	...	...	+ 123	...	(136)
	...	+ 320	...	+ 299	+ 310	+ 252	+ 262
	398	403	...	397	399	469	(485)
	481	485	...	498	488	(529)	...
	570	554	...	557	560	589	...
	...	794	...	...	794	(731) }	707
	...	946	913	941	933	872 }	(890)
	...	(1080)	...	1069	1075	(1092)	1072
	...	1214	...	1271	1243	...	(1246)
	1298	1317	...	...	1308	1311	...
	...	(1403)	...	1378	1390	...	1419
	1491	1488	...	1499	1493	...	(1509)
	1601	(1602)	...	1599	1601	1546	1599
	1697	1715	1717	1726	1714	1710	(1749)
	...	1791	...	...	1791	...	...
	1814	1839	...	...	1827	...	...
	+1882	1894	1868	1871	1879	(*noted*)	+1898

Note.—Figures in heavy type refer to maxima, ordinary type minima. Figures in brackets are calculated.

* *Pub. Ast. Soc. Pac.*, **31**, 183. † *L.O.B.*, **10**, 34

TABLE III.

The Dark Space at −450 *km./s.* ±

Date.		Plate No.	H_ϵ.	H_δ.	H_γ.	Neb. Bd. 4363.	Helium. 4472.	Harper* H_β, H_γ, H_δ.
1918.								
June	28	5281			−460			?
	29	5284			−468		−431	Corrn.
July	1	5286					−428	−20 Km.
	2	5287	−428					to be
	2	5288			−484		−444	applied
	5	5290		−433				
	6	5291			−452			
	10	5298					−476	
	15	5299		−449	−461			
	19							−455
	20							−453
	21							−448
	24							−450
	25							−456
	26							−443
	30							−439
	31							−445
Aug.	1							−442
	3	5313			−465	−497		
	3	5314			−465	−485		
	10	5321			−445			
	15	5323			−455			
	22	5329			−434			
	23							−443
	24							−434
	25							−449
	26							−443
	27							−444
	29							−451
Sept.	7							−442
	12	5346.		−434	−433	−444		
	12	5347		−450	−458			
	21							−462
	22							−447
	30							−446
Oct.	3							−440
	4	5361			−436	−454		

* *Journ. Roy. Ast. Soc. Canada*, **12**, 509.

TABLE III.—*continued.*

Date.	Plate No.	H_ϵ.	H_δ.	H_γ.	Neb. Bd. 4363.	Helium. 4472.	Harper* H_β, H_γ, H_δ.
1918.							
Oct. 8							−449
21							−447
Nov. 2							−444
10							−444
1919.							
Apr. 3–5	5443				−427		
June 9–12	5463				−408		
July 22–25	5484				−407		
Aug. 27–Sept. 1	5499				−438		
Means		−428	−442	−455	−445	−445	
Weighted Mean				−448 km./s.			−446 km./s.

TABLE IV.

The pair of Dark Lines at λλ4054, 4060 (1918 *July* 15).

	1918.	Harper.†	Lunt.	Paddock.‡	Evershed.§
I.	June 15	...	f. dark shading	59 & 64 diff. 62	
	16	...	59–67 v. indef. 63	61 & 68 ,, 65	57–67 weak, 62 diff.
	17	58–70 64	59–67 63		
	18	57–68 63	57–67 62	59 & 65 ,, 62	,, more pronod.
	19	56–68 62	...	59 & 66 ,, 63	54–64 du. bd. 59
	20	58–71 f. diff. 65	...	59 & 65 fair 62	57–67 62
	21	...	56–68 62	58 & 64 ,, 61	
	22	...	56–68 62	57 & 64 ,, 61	
	23	53–66 v. diff. 60	...	55 & 61 diff. 58	? 53–? 64 59
	24	...	...	57 & 64 fair 61	
	25	51–64, 55·1 & 60·1 58	...	54 & 59 good 57	

* *Journ. Roy. Ast. Soc. Canada*, **12**, 509.
† *Ibid.*, **12**, 497.
‡ *Pubns. Ast. Soc. Pac.*, **31**, 56.
§ *M.N.*, **79**, 479.

TABLE IV.—*continued.*

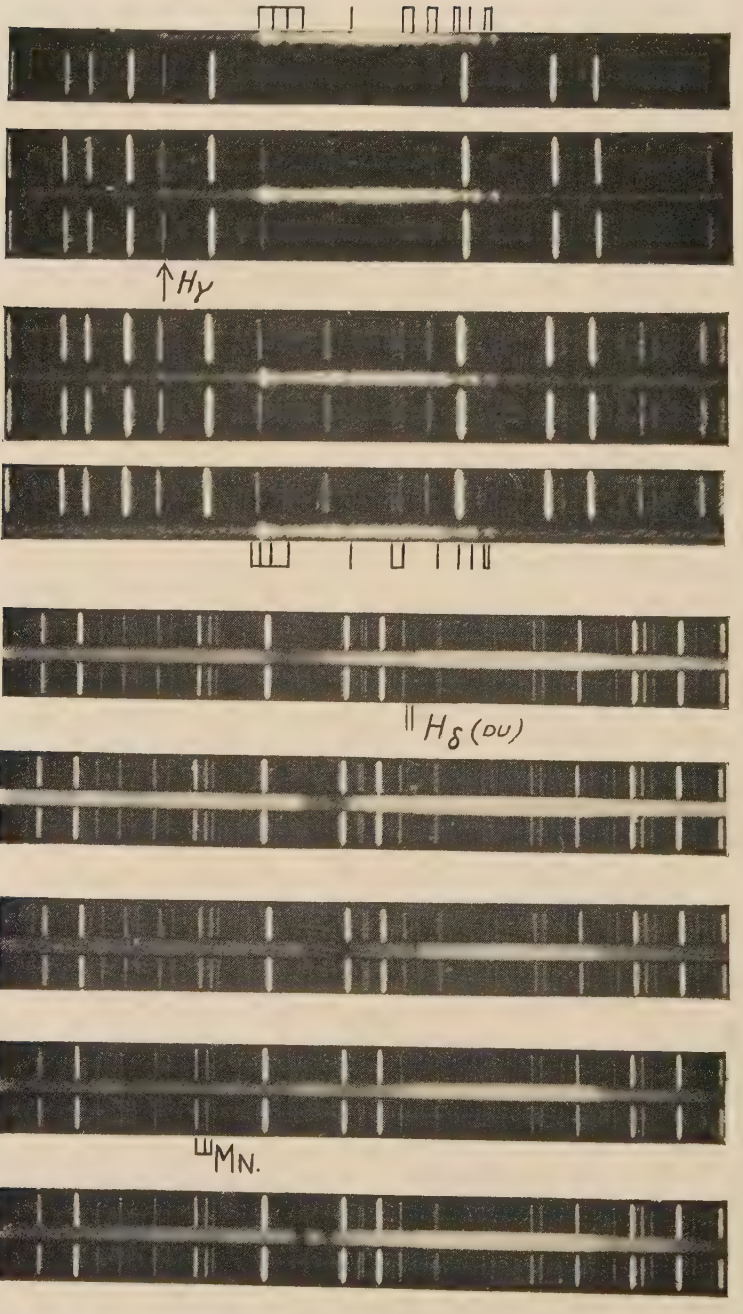
	1918.	Harper.*	Lunt.	Paddock.†	Evershed.‡
	June				
II.	26	62–75 69	...	55	
	27	...	indef. bright f.	55	centre of bright band
	28	...	indef. ,,	55	
	29	64–75 70	indef. ,, f.	55	
	July				
III.	1	...	continuous	(40 & 47) fair 44	? 53–? 64 bright 59
	2	49–64, 53 & 60 57	51 & 56 54 } 2 diff. dark lines	53 & 60 57	
	3	54–67 61	...	57 & 63 60 } strong and wide	? 53–? 64 59 } v. strong
	4	51–64 58	...	56 & 62 59	? 53–? 64 59 } dark
	5	54–66, 57·4 & 62·4 60	56 & 62 ,, ,, 59	55 & 61 58 } fairly sharp	
	6	54–[66 diff.] 60	54–63 ,, merged 59	54 & 60 sharp 57	? 53–? 64 less dark 59
	7	...	...	52 & 58 ,, 55	
	8	...	53– ? broad, indef.	...	
IV.	9	[46–53] new bd. 50	...	57	
	10	62–75 69	53–59 bright 56	57 } centre of bright band	
	11	46–75 61	...	...	nearly vanished, faint emission.
	12	65–75 70 } f. diff. 4048 ?	...	58	
	13	46–54 f. diff. 50	...	...	

* *Journ. Roy. Ast. Soc. Canada*, **12**, 497. † *Pubns. Ast. Soc. Pac.*, **31**, 56.
‡ *M.N.*, **79**, 479.

TABLE IV.—*continued.*

	1918.	Harper.*	Lunt.	Paddock.†	Evershed.‡
	July				
V.	14	49–53 (defined, narrow) 51	...	52 & 58 sharp 55	
V.	15	50–54 52	54 & 60 strong, dark 57		
V.	16	...	...	53 & 60 (sharp, strong) 57	
V.	17	51·6 & 58·2 intense 55	...	...	
	18	46–49 not pronced. 48	...	56	
	19	62–76, 48 & 53 ? 69	...	57	
	20	65–74 not pronced. 70	...	...	
	21	68–74 71	...	57	
	22	...	slight brightening	...	
	23	...	...	56	
	25	65–73 (v. f., 4048 ?) 69	...	57	
	26	63–77 v. f. 70	...	...	centre of bright band
	27	...	continuous	...	
	28	...	...	55	
	30	64–76 (also 45–52 diff.) 70	...	...	
	31	63–74 (also 46–52 diff.) 69	bright, indefinite	...	
	Aug. 1	63–75 69	...	...	
	3	...	bright, faint	...	
	5	.	...	57	
	8	..	...	54	
	11	...	...	57	

* *Journ. Roy. Ast. Soc. Canada*, **12**, 497. † *Pubns. Ast. Soc. Pac.*, **31**, 56.
‡ *M.N.*, **79**, 479.



Spectra of Nova Aquilæ.

Description of Plate.

Strips I. and II. Plate 5463. 1919 June 9, 10, 12, nebular band λ4363.

Iron spark comparison spectrum, violet to left, magnification eleven diameters. The *dark* lines marked above Strip I. are those shown in Table II. column II. First group of four (left to right) −1851, 1645, 1453, 1192. Single line near centre −408. Two wide pairs +403 and 554, 794 and 946. Close pair +1214 and 1317, followed by +1488.

The very dark space at right hand end contains a faint bright line just before terminal bright patch, dividing it into two unequal dark spaces at +1715 and +1839. The faint bright line will be lost in reproduction.

The arrow under Strip II. shows a faint diffuse bright patch due to H_γ corresponding to the brightest patch at −1748 in the nebular band. It is displaced −1740. Between this point and the nebular band, the H_γ band shows faintly.

Strips III. and IV. Plate 5499. 1919 August 27, 28, 30, September 1.

The lines under Strip IV. show the *bright* patches given in Table II. column IV.

A faint bright line (better seen in I. and II.) at −1900 precedes the brightest patch at −1756. −1581 and 1301 complete the first group of four. These are followed by a very broad patch ending at the strong dark line −438, which is followed by the maximum at −313. Beyond the centre, the two maxima flanking dark +397 are seen at +299 and 498. The dark line at +794 in Plate 5463 is wanting, leaving a broad bright patch between +557 and 941.

The broad bright patch at +1069 precedes the last three, +1378, 1599 and 1871, the terminal one having a faint companion at +1791 in Plate 5463, but not measurable in Plate 5499.

Strips V.–IX. Showing changes in the double band at λλ4054, 4060 (July 15). Plates 5287, 5289, 5290, 5297 and 5299. Dates 1918 July 2, 5, 5, 10, and 15. Magnification 7·6 diameters. V. and VII. show the difference in displacement very clearly. VII. shows that insufficient exposure results in the two bands merging into one broad dark diffuse band. VIII. shows the region bright, whilst IX. shows the pair at maximum intensity and sharpness, but even then very dark, broad and diffuse in comparison with H_δ shown further to the red. V. shows the duplicity of H_δ recorded in the second paper (*M.N.*, 80, 519) Table IV.

The oblique line on violet side of dark H_δ in VI. is a fault.

In VIII., July 10, dark H_δ is absent (see above Table).

Nova Persei No. 2 (*Anderson*). By E. E. Barnard.

This star has been under observation by the writer with the large telescope more or less closely since 1901. Absence with the eclipse expedition to Sumatra prevented observations when the nova was in its brightest stages. Since it became faint, however, it has been closely watched, especially during the last three or four years, during which time it has shown many interesting changes in brightness. Indeed, the star has not really been stationary in its light for any considerable time during that period. The greatest fluctuations seem to have been about two magnitudes (1917 Aug. 30, $12^{m}\cdot4$, 1918 Jan. 29, $14^{m}\cdot4$), though in general the light-changes have been much less than this. These variations do not seem to have a definite period, but are perhaps of an irregular nature. They are sometimes rapid. Thus from 1919 Nov. 15 to Nov. 18 it increased from magnitude 13·7 to 12·6, while in the next three days it declined from magnitude 12·6 to 13·6. Professor E. C. Pickering showed from the Harvard photographs (*H.C.O. Circular*, 66) that, previous to its outburst in 1901 Feb., the star was subject to changes of brightness of as much as a whole magnitude. It is probable, therefore, that the nova has resumed its original peculiarities of variation. A study of its present condition may help us to understand something of its physical condition in the pre-nova state.

In *A.N.*, 3796 (Bd. 159, p. 49), I have given measures, with a diagram, of 14 small stars within a radius of 3′ or 4′ of the nova. Six other stars, Nos. 15 and 20, and more recently Nos. 16, 17, 18, and 19, have been introduced. Some time ago I compared each star used on any one night with the deduced magnitudes of the nova for that night, and by this means determined the brightness of each star on my scale of magnitudes. Let c be the brightness of the comparison star. Then

$$c = \mathrm{N} \pm n$$

where n is the estimated difference of brightness of the nova and comparison star in tenths of a magnitude and N the magnitude of the nova as deduced from all the comparison stars used that night (whose magnitudes I had estimated previously), including c. By this means I have determined the magnitudes of my comparison stars. Nos. 17 and 18 are omitted because they have not been used long enough to give results of value.

Professor F. H. Seares at Mount Wilson has kindly supplied me with his photo-visual magnitudes of these stars with their color-indices. These are given as follows:—

Star.	Photo-visual Mag.	Color-Index.	Star.	Photo-visual Mag.	Color-Index.
1	15·19	+1·25	10	13·47	+1·10
2	14·53	+1·04	11	13·60	+0·84
3	13·67	+0·75	12	14·22	+1·13
4	15·01	+1·61	13	14·20	+1·41
5	13·68	+0·82	14	12·64	+0·73
6	14·74	+0·98	15	12·32	+0·88
7	14·38	+0·82	16	13·20	+0·87
8	13·16	+1·08	17	13·82	+0·89
9	13·94	+1·64	18	13·94	+0·57

Professor Seares says: "Each series of magnitudes—photographic and photo-visual—is based on five polar comparison plates, using the Mount Wilson standards. For only two of the plates does the deviation of the zero-point from the adopted mean exceed 0·03 mag., and these compensate each other, one being +0·11, and the other −0·12. The results must therefore be in close accordance with the international zero-point."

My own values as determined by the method described above, with the deviation from Seares' values, are as follows:—

Star.	Mag.	No. Nights.	B−S.	Δ.
			m	m
1	14·50	9	−0·69	+0·11
2	13·71	21	−0·82	−0·02
3	12·86	72	−0·81	−0·01
4	13·37	3	[−1·64]	
5	12·90	76	−0·78	+0·02
6	13·63	19	−1·11	−0·31
7	13·59	33	−0·79	+0·01
8	12·21	36	−0·95	−0·15
9	13·27	43	−0·67	+0·13
10	12·68	57	−0·79	+0·01
11	12·80	54	−0·80	+0·00
12	13·44	24	−0·78	+0·02
13	13·50	1	−0·70	+0·10
14	11·95	14	−0·69	+0·11
15	11·55	2	−0·77	+0·03
16	12·39	13	−0·81	−0·01
Mean (omitting No. 4)			−0·80	

Star No. 4 is omitted from the mean. It and No. 6 are discordant. There is a possibility of variability in one or both of these stars.

The last column (Δ) contains the residuals B − S after the mean deviation, −0·80, has been applied to my magnitudes.

The following table contains the observed magnitudes of the nova from my visual observations with the 40-inch telescope. They are dependent upon Seares' magnitudes of the comparison stars, and are, therefore, on his scale of magnitudes. Though these magnitudes are given to the second decimal place, it is probable that in most cases the first place alone is reliable. I believe in general they are to be trusted to perhaps the nearest tenth of a magnitude.

Visual Magnitudes of Nova Persei.

Date.	C.S.T. h	m	J.D.	Mag.	No. of Comps.
1916 Dec. 9	9	00	2421207	14·05	3
16	11	30	1214	13·93	2
21	8	00	1219	14·04	2
27	7	00	1225	13·89	4
30	13	20	1228	14·25	4
1917 Jan. 16	10	00	1245	13·33	3
23	9	10	1252	12·85	5
Feb. 10	11	00	1270	13·34	4
20	6	45	1280	13·72	4
27	7	00	1287	13·96	4
Mar. 27	7	30	1315	12·97	3
Aug. 18	14	00	1459	13·78	3
25	13	00	1466	13·28	6
29	15	00	1470	12·51	4
30	13	30	1471	12·38	4
Sept. 5	15	35	1477	12·85	5
25	11	50	1497	13·04	2
27	12	40	1499	13·22	4
Oct. 5	15	40	1507	13·22	5
6	14	35	1508	13·72	6
10	14	10	1512	14·16	8
13	11	15	1515	13·61	7
24	14	45	1526	13·84	8
27	9	35	1529	13·10	5
Nov. 1	11	20	1534	13·19	6
3	8	50	1536	13·94	6
6	8	10	1539	13·20	5
7	15	5	1540	13·88	6
8	11	5	1541	13·73	6
10	8	20	1543	13·49	6
15	10	25	1548	14·01	9
17	7	40	1550	14·20	7

Visual Magnitudes of Nova Persei—continued.

Date.	C.S.T. h m	J.D.	Mag.	No. of Comps.
1917 Nov. 20	8 15	2421553	13·70	7
Dec. 1	7 40	1564	14·22	6
5	14 35	1568	14·26	6
6	11 00	1569	14·11	9
8	7 25	1571	13·77	10
29	5 35	1592	14·39	8
1918 Jan. 8	5 55	1602	14·29	9
17	8 50	1611	14·05	7
19	9 20	1613	14·10	8
22	8 25	1616	14·29	8
29	7 55	1623	14·49	3
Feb. 9	8 45	1634	14·27	5
Mar. 2	8 50	1655	14·08	8
Aug. 27	11 40	1833	13·47	8
Sept. 7	12 30	1844	13·20	6
12	12 25	1849	13·48	6
19	12 25	1856	14·05	6
21	13 10	1858	13·95	6
26	12 25	1863	13·67	6
28	12 00	1865	14·12	8
Oct. 3	12 20	1870	14·07	5
5	10 40	1872	13·71	7
8	11 50	1875	14·06	7
12	10 35	1879	13·84	6
15	10 5	1882	13·72	6
29	11 20	1896	14·15	5
Nov. 5	9 00	1903	13·36	5
12	9 40	1910	12·80	5
23	8 25	1921	13·35	6
26	8 40	1924	13·26	5
30	8 5	1928	13·41	5
Dec. 3	11 35	1931	13·78	5
5	12 50	1933	13·44	5
7	7 55	1935	13·48	5
14	8 15	1942	13·44	5
17	8 40	1945	12·97	3
1919 Jan 11	7 00	1970	14·22	6
14	6 50	1973	14·34	7
18	9 20	1977	13·57	7

Visual Magnitudes of Nova Persei—continued.

Date.	C.S.T. h	m	J.D.	Mag.	No. of Comps.
1919 Jan. 25	8	00	2421984	14·27	8
28	8	25	1987	14·21	9
30	10	20	1989	14·13	9
Feb. 1	7	25	1991	14·01	9
4	7	35	1994	14·09	9
11	7	15	2001	14·33	10
15	7	15	2005	14·38	10
18	10	00	2008	14·21	9
25	8	15	2015	14·24	10
Mar. 1	7	10	2019	13·40	6
18	8	20	2036	14·13	9
22	8	10	2040	14·12	10
Apr. 1	8	00	2050	14·06	11
8	7	35	2057	14·17	9
Sept. 11	13	45	2213	12·93	8
16	13	00	2218	13·04	7
23	12	20	2225	13·34	6
27	11	45	2229	13·12	6
Oct. 2	12	50	2234	13·27	7
7	11	00	2239	13·24	6
11	10	35	2243	12·87	7
14	10	50	2246	12·92	7
16	12	40	2248	12·88	7
18	10	00	2250	12·88	7
21	10	00	2253	12·76	7
23	10	20	2255	12·76	8
28	10	5	2260	12·91	7
Nov. 1	10	00	2264	12·86	7
4	9	30	2267	12·92	7
11	8	15	2274	13·19	8
15	9	5	2278	13·74	8
18	9	15	2281	12·61	7
22	8	45	2285	13·61	8
Dec. 2	6	50	2295	13·59	8
4	11	00	2297	14·01	6
9	8	50	2302	13·40	10
11	10	20	2304	13·16	8
13	7	35	2306	12·67	8
30	5	55	2323	12·74	8

Visual Magnitudes of Nova Persei—continued.

Date.	C.S.T. h m	J.D.	Mag.	No. of Comps.
1920 Jan. 10	7 5	2422334	13·58	9
13	5 55	2337	13·17	8
13	6 30	2337	13·28	6
17	6 00	2341	14·03	13
20	6 5	2344	13·32	9
24	6 00	2348	13·22	10
Feb. 10	6 15	2365	13·13	9
28	9 50	2383	13·80	13
Mar. 6	7 50	2390	13·95	13
9	8 30	2393	13·29	8
13	8 25	2397	13·96	10
20	8 40	2404	13·22	8
21	8 5	2405	13·40	7

The times are 6^h 0^m slow of Greenwich mean time.

In the last column, "No. of Comparisons," the number in most cases indicates the number of comparison stars used, though sometimes combinations were used. The details of these observations are reserved for later publication.

The individual position measures of the early observations were printed in *A.N.*, 3796, 3829, 3888, and 4220. Those made since then are given here in detail. For comparison I have collected the means of all the various sets of measures and give them here in tabular form. They do not show any certain change in the position of the nova in the past seventeen or eighteen years. Star No. 1 was very difficult in the early measures because of the brightness of the nova at that time. I do not, therefore, place any confidence in the reality of the apparent increase of distance between it and the nova. The later measures are undoubtedly the best. Most of these stars are rather faint for the best micrometer work—especially when the conditions of seeing are not good.

Following are the more recent measures:—

Nova and 1.

		°	″
1907·758	Oct. 4	63·69	20·32
·763	,, 6	64·81	20·49
1907·760		64·25	20·40
1917·784	Oct. 13	65·30	20·67
·841	Nov. 3	66·83	20·68
·849	,, 6	67·34	...
·855	,, 8	66·35	19·58
·861	,, 10	66·72	20·27
·874	,, 15	65·66	20·22
1917·844		66·40	20·28

Nova and 2.

		°	″
1916·973	Dec. 21	107·07	91·03
·990	,, 27	106·71	91·07
1916·981		106·89	91·05

Nova and 3.

		°	″
1914·093	Feb. 3	116·48	102·61
1916·973	Dec. 21	116·12	102·81
·990	,, 27	116·04	103·05
1917·650	Aug. 25	116·49	102·66
·663	,, 30	116·29	102·61
1916·674		116·28	102·75
1919·777	Oct. 11	116·45	102·56
·791	,, 16	116·66	102·82
·804	,, 21	116·33	102·73
·810	,, 23	116·24	102·49
·996	Dec. 30	116·23	102·66
1919·836		116·38	102·65

Nova and 5.

		°	″
1916·973	Dec. 21	21·04	124·31
·990	,, 27	21·17	124·63
1917·650	Aug. 25	21·31	124·23
·663	,, 30	21·18	124·13
·850	Nov. 6	21·39	124·13
1917·452		21·22	124·29
1919·777	Oct. 11	21·36	124·05
·791	,, 16	21·31	124·06
·796	,, 18	21·38	124·31
·810	,, 23	21·20	124·04
·862	Nov. 11	21·21	124·08
1919·807		21·29	124·11

Nova and 8.

1916·990	Dec. 27	63°·15	158″·69
1917·650	Aug. 25	63·17	158·44
1918·685	Sept. 7	63·14	158·94
·698	,, 12	63·36	158·92
·737	,, 26	63·19	158·76
·742	,, 28	63·29	158·76
1918·250		63·22	158·75
1919·777	Oct. 11	63·15	158·92
·791	,, 16	63·13	159·16
·804	,, 21	63·12	159·01
·996	Dec. 30	63·11	159·17
1919·842		63·13	159·07

Nova and 10.

1919·791	Oct. 16	114°·31	169″·66
·804	,, 21	114·29	169·84
·810	,, 23	114·35	169·48
1919·802		114·32	169·66

Nova and 15.

1917·661	Aug. 29	251°·21	277″·25
·663	,, 30	251·20	277·59
1917·662		251·20	277·42

Nova and 16.

1919·739	Sept. 27	66°·02	239″·03

Nova and 20 ($16\frac{1}{2}$ *mag.*).

1917·841	Nov. 3	336°·69	46″·64
·874	,, 15	338·37	46·69
1917·857		337·53	46·66

3 *and* 10.

1916·979	Dec. 23	111°·42	67″·25

5 *and* 18.

1920·045	Jan. 17	239°·35	140″·55
·064	,, 24	239·49	140·99
1920 054		239·42	140·77

8 *and* 16.

1920·034	Jan. 13	71°·49	80″·54
·045	,, 17	71·38	80·36
·054	,, 20	71·73	80·10
·064	,, 24	71·50	80·21
1920·049		71·52	80·30

8 *and* 17.

1920·034	Jan. 13	55°·33	102′·71
·045	,, 17	55·67	102·21
·054	,, 20	55·65	102·37
064	,, 24	55·36	102·30
1920·049		55·50	102·40

15 *and* 19 (14½ *mag.*).

1920·034	Jan. 13	93°·66	8″·41
·054	,, 20	94·97	9·27
1920·044		94·33	8·84

In the following table the means of the observations of the different years have been collected :—

Nova and 1.

1901·920	66°·59	19″·44	5 n.
1906·304	66·84	19·82	2
1907·760	64·25	20·40	2
1917·844	66·40	20·28	6–5

Nova and 2.

1901·967	107°·42	90″·67	7
1916·981	106·89	91·05	2

Nova and 3.

1901·907	116°·35	102″·61	7
1902·898	116·41	102·55	4
1907·155	116·35	102·61	7
1916·674	116·28	102·75	5
1919·836	116·38	102·65	5

Nova and 4.

1902·059	142°·77	120″·57	4

Nova and 5.

1901·887	211°·14	124″·37	6
1902·967	211·21	124·20	3
1907·155	211·10	124·34	7
1917·425	211·22	124·29	5
1919·807	211·29	124·11	5

Nova and 6.

1901·960	56°·79	132″·02	4

Nova and 7.

1901·999	77°·92	132″·35	6

Nova and 8.

1901·937	63°·10	158″·45	8
1902·967	63·00	158·70	3
1907·394	62·89	158·55	3
1918·250	63·22	158·75	6
1919·842	63·13	159·07	4

Nova and 9.

1901·929	303°·50	164″·78	6
1903·032	303·61	164·70	3

Nova and 10.

1901·998	114°·51	169″·59	7
1919·802	114·32	169·66	3

Nova and 11.

1901·952	292°·95	181″·59	7
1902·770	292·93	181·67	5

Nova and 12.

1902·021	113°·00	190″·54	5

Nova and 13.

1901·965	352°·48	193″·07	3

Nova and 14.

1902·063	300°·62	236″·47	5
1903·052	300·40	236·47	2

Nova and 15.

1917·662	251°·20	277″·42	2

Nova and 16.

1919·739	66°·02	239″·03	1

8 *and* 17.

1920·049	55°·50	102″·40	4

8 *and* 16.

1920·049	71°·52	80″·30	4

5 *and* 18.

1920·054	239°·42	140″·77	2

Nova and 20.

1917·857	337°·53	46″·66	2

3 *and* 10.

1916·979	111°·42	67″·25	1

15 *and* 19.

1920·044	94°·33	8″·84	2

In *Lick Observatory Bulletin*, No. 8, Professor Tucker, from numerous meridian-circle observations, gives an accurate position of Nova Persei:

$$1901{\cdot}0\ \alpha\ 3^{h}\ 24^{m}\ 28^{s}{\cdot}086,\quad \delta + 43^{\circ}\ 33'\ 54''{\cdot}03.$$

In the same number of the Bulletin Professor Aitken gives a list of his micrometer measures (with a chart) of the nova and a number of small stars near it, when measures were difficult on account of the brightness of the nova. Most of the stars were also observed by me later, when the new star had lost much of its brightness. Those stars, that are common to both observers, are arranged in the following table for identification:—

A.		B.	A.		B.
b	=	2	*j*	=	9
c	=	3	*k*	=	10
e	=	4	*l*	=	11
f	=	5	*m*	=	12
g	=	6	*n*	=	13
h	=	7	*o*	=	14
i	=	8			

The star B, No. 20, is the nearest that I could find to Aitken's *a*.

On 1916 December 16, when the nova was faint, the writer found (visually) a nebulosity which could be distinctly seen connected with it closely preceding and involving the star. This nebulosity, which was later verified at Mount Wilson by Pease and Ritchey by photography, has been seen since at all observations when the moon and atmospheric conditions would permit. It will depend on photography to show if any change takes place in

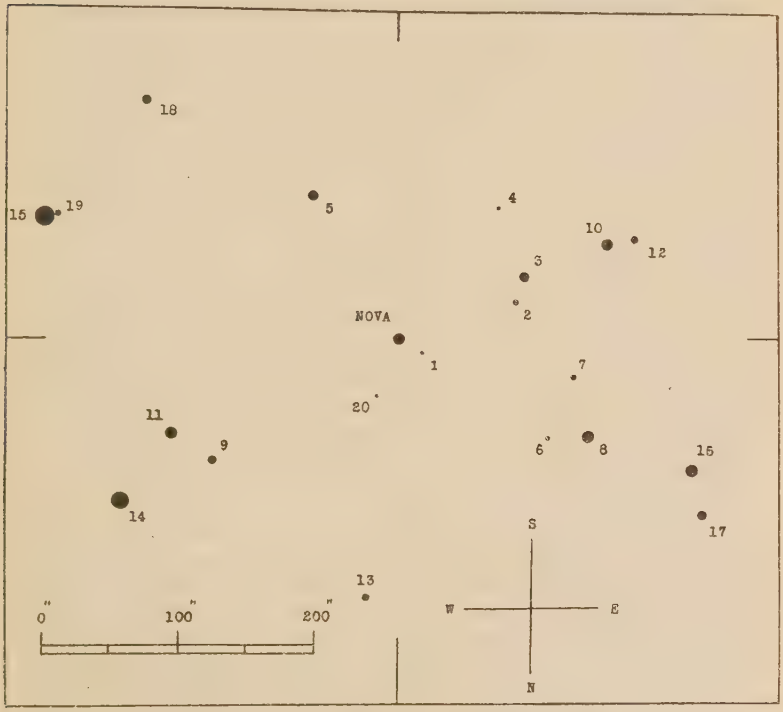


Chart of small Stars near Nova Persei of 1901.

this object, which is undoubtedly connected with the star and may have had something to do with the outburst. One of the photographs at Mount Wilson shows that part of the nebulosity surrounds the star symmetrically, giving it the appearance of a planetary nebula with the nova as a nucleus. This is important in connection with Wright's discovery that the novæ, Wolf-Rayet stars, and the nuclei of the planetary nebulæ are closely related in their spectra. What is seen in the 40-inch telescope is not the circular disc of nebulosity shown on the photograph, but the narrow strip closely preceding the star and involving it.

The nova seems to have returned to its original condition of

brightness several years ago, and, as we have seen, it has resumed its original activity as a small variable star. Though inferior to Nova Aquilæ III. (1918) at its greatest brightness, it started from a lower magnitude—at least two magnitudes fainter than Nova Aquilæ,—and its total increase of brightness was decidedly greater than of that star, so that had its original condition been as bright as the latter star it would have been at its maximum a brighter object than Nova Aquilæ.

It is altogether probable that these stars return very closely to their original brightness, and probably also to their original physical condition. The interval between the outburst and the return to the normal state is comparatively brief, ranging through eight or ten or fifteen years. Their sudden increase of brightness covers a few hours, or days at most. In general it takes something like a dozen years or so for them to return to the normal. Nova Lacertæ (Espin) of 1910, in its decline, reached its former brightness several years ago. Nova Geminorum I. (Turner) of 1903 is perhaps still fading.

Some of these stars appear to have been variables to begin with, as in the present case and probably in the case of Nova Aquilæ. Nova Lacertæ shows no fluctuations in its light, though carefully watched since the return to its normal condition. Its light is perfectly steady. Like several other novæ, it is ill defined and hazy and requires a longer focus than the ordinary star. This is also the case with Nova Cygni (1876), which fluctuates in its light, Nova Aurigæ (1891), and Nova Sagittarii (1898), which require an increase of focus and which even then are not well defined. Nova Persei does not partake of this peculiarity. In general it appears like an ordinary star.

It will be seen that there are considerable differences in the novæ. Especially is this so on their return to normal condition. Some are variable and some are hazy, and require a different focus from that of a normal star. Some return rather quickly to their normal condition, while others are slower. The actual increase of brightness in some is very much greater than in others. Some are apparently connected with nebulosity, while others do not (visually or photographically) show this peculiarity.

Yerkes Observatory,
Williams Bay, Wisconsin:
1920 *April* 16.

P.S.—I have recently determined the magnitudes of the stars Nos. 17 and 18 by the same method used for the other comparison stars. I give these, here, as a continuation of the table on p. 718 :—

Star.	Mag. M	No. nights.	B—S. M	Δ. M
17	12·82	8	−1·00	−0·20
18	13·14	5	−0·80	0·00

B—S, of course, means Barnard minus Seares.

E. E. B.

1920 *July* 19.

Radiation-pressure in Solar Phenomena. By A. S. Eddington, M.A., M.Sc., F.R.S.

Until recently the great possibilities of radiation-pressure as an agent in cosmical phenomena were scarcely appreciated. Now, however, there is a tendency to go to the opposite extreme, and to invoke its aid almost too freely.

In attempting to discuss the effects of radiation-pressure on coronal and prominence phenomena, we are faced by the difficulty that it is impossible to make any close estimate of its value, owing to ignorance of the absorptive powers of the materials concerned under conditions unattainable in the laboratory. This vagueness may have encouraged quite exaggerated ideas of the possible effects of radiation-pressure in the sun's atmosphere. The object of this note is to point out that there is an upper limit to the power of radiation-pressure to support or drive out matter; and this upper limit is easily calculable. We may not be able to state exactly what it *can* do, but we can state what it *cannot* do.

Taking Abbot's value of the solar constant, 1·93 gram-calories per sq. cm. per min., we find that the amount of energy issuing from each sq. cm. of the sun's surface is $6{\cdot}2 \cdot 10^{10}$ ergs per sec. To obtain the pressure, we must divide by the velocity of light $(3 \cdot 10^{10})$; thus the momentum of the outflowing radiation is equivalent to a pressure of 2 dynes per sq. cm., and that pressure would be exerted on any surface which stops the stream.

The effect of radiation-pressure in the sun's atmosphere may therefore be compared to an upward wind of strength 2 dynes per sq. cm. It is obvious that the effects, if any, of so trifling a force will be very limited. The exact effect on any kind of material depends on its power of absorption—of stopping the wind, instead of letting it blow through. But in any case the utmost weight of material that can be supported is 2 dynes per sq. cm. area.

The pressure here mentioned is that of the radiation which finally emerges, after passing all obstruction on the sun; but we are really concerned with the pressure of the initial stream which leaves the photosphere. This may be slightly higher, but it is of the same order of magnitude. It can be calculated from the absolute temperature of the photosphere, which is about 6000° C. We shall allow an ample margin for observational uncertainties and take the initial pressure to be 30 dynes, which would correspond to a photospheric temperature of about 10,000°. The weight of a gram of matter on the sun is 26,700 dynes. Consequently the pressure of radiation cannot carry a total weight of more than a milligram per sq. cm.

Consider, for example, a cloud of prominence matter 10,000 km. deep. If it is supported by radiation-pressure, its mass cannot exceed 1 milligram per sq. cm. section, and consequently its density must be less than 10^{-12}. I suppose that this is a *reductio ad absurdum*, though I do not know of any definite reason against

this extreme tenuity. It seems unlikely that the interposition of a milligram of matter per sq. cm. can produce the dark prominence-markings on the sun's disc shown on spectroheliograms, yet it is perhaps not impossible. But I think it is usually supposed that much greater quantities of matter are involved in these outbursts; if so, we must dismiss the idea that radiation-pressure plays any appreciable part.

Similarly the corona cannot be supported by radiation-pressure unless its density is of order 10^{-15} at the most.

If the repulsion of a comet's tail is due to radiation-pressure, its average density must be of order 10^{-15} or less. The distance of the comet from the sun is immaterial, since weight and radiation-pressure both vary as the inverse square. In this probably no objection will be felt against the necessary tenuity.

Note on the Distribution of Solar Faculæ.
By E. Walter Maunder.

In 1907 May, Sir W. H. M. Christie, K.C.B., then Astronomer Royal, communicated to this Society a paper by Mrs. Walter Maunder, entitled "An Apparent Influence of the Earth on the Numbers and Areas of Sunspots in the Cycle 1889–1901," which showed that there was "a well marked and steady preponderance of the eastern half of the Sun's disc over the western half, both as to the areas of the spots and as to the numbers of the separate groups" (*M.N.*, **67**, 451–476). Towards the end of this paper the author remarked that "It is quite conceivable that there may be some physical characteristic of sunspots, purely solar in its nature, which renders them more easily visible when they are approaching us than when they are receding. Thus, if we suppose that sunspots are holes in the photosphere of which the axes are generally sloped downwards from west to east, or if we suppose that there is a great heaping up of the surface behind the sunspot, we might have some explanation of the predominance of the eastern hemisphere both in the numbers and areas of sunspots. Neither of these suggestions would, in my opinion, account for the relations which I have put in evidence, and there is another class of phenomenon which seems to show the same predominance of the eastern hemisphere over the western, which could hardly be affected in the same way" (*ibid.*, 472).

The last sentence referred to the prominences, which, unlike spots, rise to great heights above the solar photosphere, and it was pointed out that M. Sykora, in a note published in 1897, had drawn attention to the circumstance that there was a slight but unmistakable predominance of eastern prominences. Continuing their summation to the end of 1905, as derived from the Catania observations, it was shown that from 1892 to 1900 inclusive the

eastern hemisphere was the more prolific, but with the year of minimum, 1901, the tide turned, and for four years the western had the advantage. In 1905 the east had recovered its earlier predominance. In more recent years the watch on the oscillations of prominence frequency between the east and west limbs of the Sun has been carried on by Mr. Evershed at Kodaikánal, who has found that, as a general rule, the eastern prominences are in excess during the years of maximum solar activity, and those of decline; during the years of minimum, and perhaps while the solar activity is increasing, the western prominences tend to be in the majority, but not to a very marked extent.

Both Mrs. Maunder and myself recognised, when her paper was written, that not only the prominences but the faculæ also might throw light upon the "apparent Earth effect." For faculæ are elevations of the solar surface, and if an eastern predominance occurred with them, it could not be ascribed to their being holes sloping backwards, or to the heaping up of the solar surface behind them. If there appeared to be more and greater spots on the eastern half of the Sun than on the western *because* the faculæ followed them and masked them somewhat during their retreat down the western slope of the Sun as seen from the Earth, then certainly the same explanation could not hold good for the faculæ if they also should seem to be in excess on the east. Still less could an eastern excess be explained by alleging that they, the faculæ, were "holes sloping backwards."

Accordingly, towards the end of 1907 I began an investigation of the distribution of faculæ, and had made some considerable progress when circumstances compelled me to put the work aside. But last year some members of the British Astronomical Association very generously expressed their willingness to help me in any simple calculations which I might wish to undertake, and by their kind assistance I have now been able to draw up the statistics set forth in the present paper. I desire, therefore, to express my great indebtedness to Messrs. R. M. Fry, Anthony Johnson, H. W. Raisin, B. G. Ronalds, D. R. Sharpe, and H. C. Trent, and to the late Mr. Bruce H. Clarke—members of the Solar Section of the British Astronomical Association—who carried out much the greater part of the tedious and fidgety work of going through the volumes of the *Greenwich Photo-Heliographic Results*, and separating the faculæ observed on each day according to the quadrants in which they appeared. These four quadrants—N.W., S.W., N.E., S.E.—are divided by the four points where the Sun's axis and its equator meet the circumference. When this separation had been made for each day, and the total areas of the faculæ in each quadrant had been ascertained, the sums of the areas were taken for each month.

As it is very easy, occasionally, to slip a group of faculæ into the wrong quadrant, it was necessary to go over every entry a second time, independently, and to examine every figure, especially with regard to the distribution of the faculæ between the east and

west limbs. Mr. Johnson and Mr. Ronalds gave me much help in this portion of the work also.

The Greenwich photo-heliographic record began in 1874, and is therefore now in its 47th year. The amount of material to be

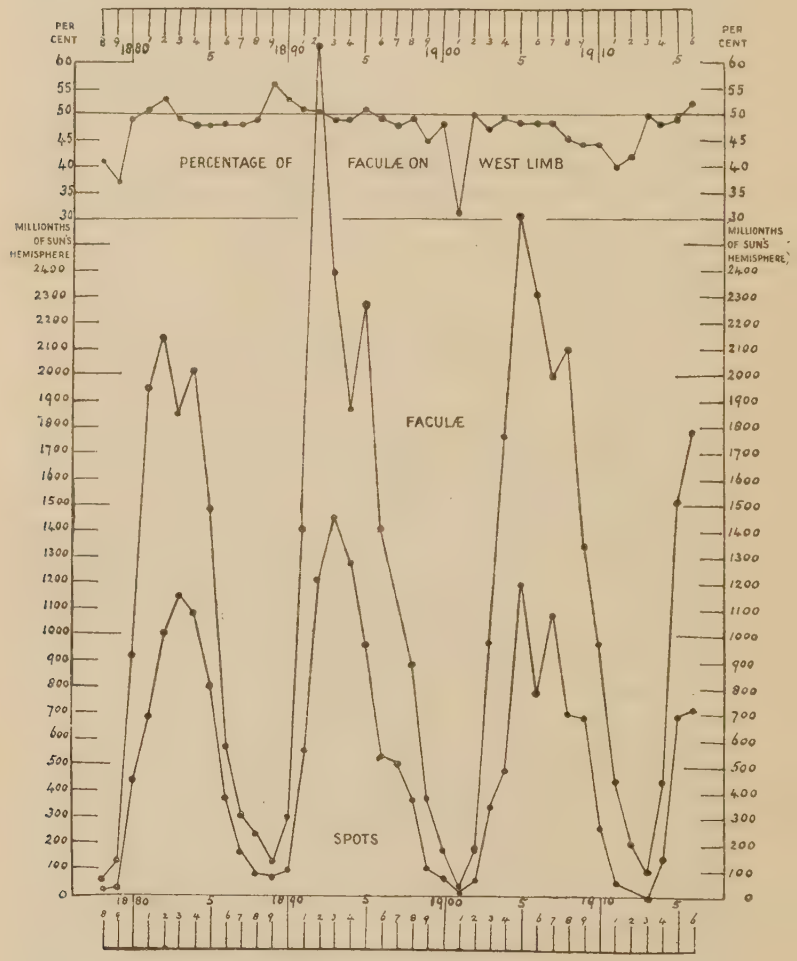


FIG. 1.

discussed is very large, and has become more than I could have hoped to deal with single-handed in any reasonable time. That I am now able to present this Note to the Society is thus wholly due to the very large part in the work which the seven gentlemen just named carried out so carefully on my behalf.

The first twelve years of the Greenwich record are not included

in the Tables of this Note, for the records for 1874 to 1877 were very imperfect, and those for 1878 to 1885 lacked the uniformity established later. The figures given in the Tables of the present

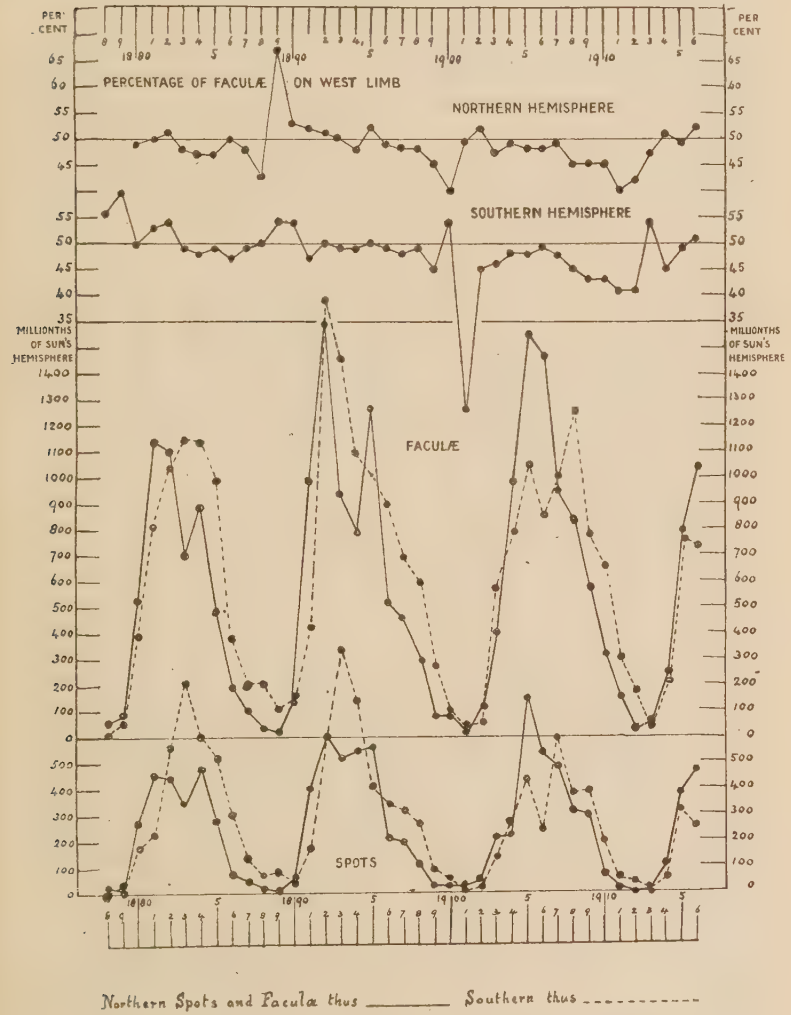


FIG. 2.

Note are therefore restricted to the years from 1886 onward, but I have made considerable progress in treating 1878–1885, and have used provisional numbers for them in the two diagrams.

Table I. supplies for the 30 years, 1886–1915, the mean daily areas of spots and faculæ, corrected for foreshortening and expressed

TABLE I.

Mean Daily Areas, North and South, of Sunspots and Faculæ, with the Proportion of the Areas of Faculæ seen on the West Limb, for each Year from 1886 *to* 1916.

Year.	No. of Days.	Sunspots.				Faculæ.						
		N.	S.	N.+S.	$\frac{100\,N.}{N.+S.}$	N.	S.	N+S.	$\frac{100\,N.}{N.+S.}$	$\frac{1000\,W.}{W.+E.}$ N.	S.	N.+S.
1886	363	76	305	381	20	191	388	579	33	499	474	482
7	361	44	135	179	25	103	201	304	34	476	489	485
8	359	20	69	89	22	37	202	239	16	430	503	492
9	360	5	73	78	6	20	111	131	15	672	536	561
1890	361	53	46	99	53	143	160	303	47	530	534	532
1	363	400	169	569	70	991	424	1415	70	520	467	504
2	362	607	607	1214	50	1587	1684	3271	49	512	502	507
3	360	517	941	1458	35	938	1466	2404	39	497	489	491
4	364	543	739	1282	42	788	1089	1877	42	481	492	488
5	364	565	409	974	58	1266	1012	2278	56	522	503	514
6	364	203	340	543	47	518	893	1411	37	487	493	491
7	364	196	318	514	38	456	690	1146	40	477	482	480
8	363	110	265	375	29	296	595	891	33	483	487	486
9	364	23	88	111	21	73	264	337	22	446	448	448
1900	360	26	49	75	35	79	101	180	44	402	546	482
1	359	22	7	29	77	13	17	30	43	483	177	307
2	349	42	21	63	67	118	60	178	66	520	446	495
3	350	132	208	340	39	397	574	971	41	476	457	465
4	363	268	220	488	55	985	783	1768	56	489	483	487
5	364	750	441	1191	63	1557	1056	2613	60	479	485	485
6	364	539	239	778	69	1467	853	2320	63	478	491	483
7	365	489	593	1082	45	948	1051	1999	47	490	475	482
8	366	316	381	697	45	837	1262	2099	40	452	452	452
9	364	299	393	692	43	575	780	1355	42	454	432	442
1910	365	66	198	264	25	317	654	971	33	434	451	445
1	364	17	47	64	27	152	307	459	33	401	397	399
2	365	1	36	37	2	34	176	210	16	414	414	414
3	365	5	2	7	67	53	42	95	56	468	536	498
4	365	99	53	152	65	239	216	455	53	506	454	481
5	365	379	318	697	54	772	748	1520	51	491	488	490
6	365	468	255	723	65	1046	737	1783	59	518	512	515

Note.—The figures for 1916 are only provisional.

The areas of Spots and Faculæ in this and the following tables are corrected for foreshortening, and are expressed in millionths of the Sun's visible hemisphere.

in millionths of the Sun's visible hemisphere. 1914 is the last year for which the *Results* have been published, but those for 1915 have already been passed for press, and provisional figures are available for 1916. The areas both of spots and faculæ are given, first for spots or faculæ north of the Sun's equator, then for those south, then for the entire disc. The percentage of the spots or faculæ of the northern hemisphere follows next. The proportion per thousand of the total faculous area is exhibited in the last three columns, (*a*) for northern latitudes, (*b*) for southern, and (*c*) for all latitudes respectively.

Tables II. and III. give the mean daily areas of faculæ for each month of the 30 years of observation, together with the difference between the areas on the west limb and those on the east; Table II. being devoted to the northern hemisphere, Table III. to the southern.

Table IV. exhibits the mean daily areas of faculæ in all latitudes for each month of the 30 years, and the totals for the 30 years are taken for each month.

The two diagrams illustrate Table I.; the first showing the mean daily area for each year of the spots and faculæ in all latitudes, together with the percentage of faculæ seen on the west limb. In the second diagram, the same particulars are exhibited as in the first, but the northern and southern hemispheres are separated.

In Mrs. Maunder's paper in May 1907 her method of investigation was to divide the Sun's apparent disc into 14 lunes, and to take the sums of the spotted areas in each lune for the whole cycle considered. She did not tabulate the results according to months or to years.

In my present paper I have followed a somewhat different method, partly of necessity. Since the faculæ are very rarely distinguishable when near the centre of the disc and do not present identifiable characteristics throughout their passage across the Sun, as groups of sunspots do, it is not possible to follow out the history of a group of faculæ from day to day or from one rotation to another with the same certainty and ease as we can in the case of spot groups. Further, as they are so large and irregular, their heliographic longitudes and latitudes cannot be determined with the precision possible in the case of spots. Faculæ are most distinct near the Sun's limb, east and west, when they are most foreshortened and therefore badly presented. When they appear to cover the greatest areas, and are presented best, they become merged with the general surface, and part of a facula may be distinguished when another part is not.

The three orders of phenomena are therefore necessarily studied under different conditions, and the measurement of their areas has a different signification. Spots, which may be considered for this purpose as practically stains on the solar surface, can be watched in their progress across the disc from the east limb to the west; faculæ, generally speaking, can be best seen near the limb, when

TABLE II.

Mean Daily Areas of Faculæ (NORTHERN *Hemisphere*), *with the Mean Differences in Area between the West and East Limbs, for each Month, from* 1886 *to* 1915.

Month.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.
	1886.		1889.		1892.		1895.		1898.	
Jan.	341	+ 7	3	+ 3	1588	+336	1035	+ 91	830	− 2
Feb.	154	+ 8	6	− 6	1577	+157	951	+103	437	− 29
Mar.	194	−24	30	+ 28	1614	+124	1207	− 31	250	− 14
April	148	−16	51	+ 3	1785	+ 65	1379	+103	176	−102
May	301	−31	0	0	1822	−108	1312	+228	188	− 30
June	196	−30	5	− 3	1654	− 42	1383	+ 85	191	+111
July	189	−11	18	+ 14	1485	−269	1077	+ 67	135	− 3
Aug.	233	+ 5	6	+ 2	1555	+131	1467	− 55	245	− 21
Sept.	256	+18	4	− 4	1556	− 42	1540	−152	206	− 46
Oct.	94	+10	42	− 8	1845	− 49	1453	−191	353	− 33
Nov.	83	+17	23	+ 5	1305	+151	1246	+306	415	+ 75
Dec.	113	+41	47	+ 47	1263	+ 9	1127	+121	132	− 26
	1887.		1890.		1893.		1896.		1899.	
Jan.	113	−17	110	+ 36	908	− 28	704	+ 40	34	+ 4
Feb.	142	−60	53	+ 11	790	+ 92	557	+ 5	12	− 12
Mar.	220	− 8	207	− 65	937	+ 7	568	− 96	96	− 24
April	125	−23	34	+ 14	841	−113	674	+ 6	77	− 33
May	139	−29	87	+ 27	1034	+ 34	400	+ 90	28	− 2
June	97	+ 3	10	+ 4	1184	+ 72	264	+ 42	160	− 56
July	73	−25	53	− 39	1065	− 87	364	− 54	229	+ 51
Aug.	101	+ 7	78	− 34	1148	+ 8	264	+ 16	57	− 23
Sept.	67	+63	266	+108	843	+ 67	731	− 35	57	− 37
Oct.	57	+ 5	288	+ 26	935	+ 39	564	− 88	40	+ 20
Nov.	91	+43	219	− 55	609	− 51	484	−128	23	+ 19
Dec.	18	−18	308	+ 68	940	−102	646	+ 38	60	− 2
	1888.		1891.		1894.		1897.		1900.	
Jan.	81	−49	148	− 64	563	− 23	405	− 49	91	− 69
Feb.	33	− 3	399	+ 25	707	+ 45	747	−131	280	− 54
Mar.	83	−19	498	− 98	583	− 51	691	+111	61	+ 13
April	24	0	523	+ 95	746	+ 72	731	+149	121	− 47
May	13	+ 1	762	+ 12	929	−277	517	− 37	228	− 28
June	63	+37	1286	+124	794	+ 90	175	− 17	80	+ 16
July	40	−26	1774	+272	1116	− 24	175	− 15	45	− 15
Aug.	2	+ 2	1248	− 32	803	− 11	201	− 5	9	− 3
Sept.	23	− 9	1341	−197	829	− 13	517	−211	52	− 4
Oct.	5	− 5	1363	+ 55	1049	− 21	370	− 8	0	...
Nov.	55	+11	1316	+134	605	−149	171	+ 37	9	+ 9
Dec.	24	− 2	1140	+156	712	+ 10	808	− 80	10	− 10

TABLE II.—*continued.*

Mean Daily Areas of Faculæ (NORTHERN *Hemisphere*), *with the Mean Differences in Area between the West and East Limbs, for each Month, from* 1886 *to* 1915.

Month.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.
	1901.		1904.		1907.		1910.		1913.	
Jan.	0	...	548	+ 84	1620	−126	426	− 64	137	+ 7
Feb.	0	...	656	+ 70	1550	− 20	336	− 94	85	− 15
Mar.	14	− 2	780	−108	1385	+ 73	372	− 12	84	− 28
April	0	...	899	−165	997	− 57	268	+ 10	32	− 24
May	63	+ 11	951	+ 39	585	− 65	360	− 74	24	+ 16
June	37	− 11	954	− 92	471	− 93	322	− 20	18	− 10
July	21	− 5	1137	−273	521	− 13	389	− 37	52	+ 26
Aug.	6	− 6	1113	+ 7	575	− 1	248	− 68	21	− 17
Sept.	3	+ 3	773	− 83	931	− 39	284	−156	55	+ 3
Oct.	0	...	1088	+132	957	+ 81	309	+ 45	42	− 4
Nov.	0	...	957	+ 45	906	+134	266	− 16	46	+ 14
Dec.	5	+ 5	1908	+ 74	926	−110	214	− 16	43	− 9
	1902.		1905.		1908.		1911.		1914.	
Jan.	28	− 14	1321	−141	1141	−251	136	+ 38	66	+ 20
Feb.	2	+ 2	1448	0	1140	+ 58	101	+ 57	57	+ 49
Mar.	173	+ 35	1314	−314	723	− 85	133	+ 15	115	+ 7
April	131	− 47	1393	−199	467	+ 53	173	− 57	340	− 64
May	173	−103	1467	+119	472	− 14	118	− 46	358	+ 86
June	182	+ 98	1428	− 14	537	− 81	123	− 13	326	− 20
July	18	+ 12	1496	+ 24	614	− 40	173	− 55	49	+ 3
Aug.	38	− 16	1862	− 96	939	−149	257	− 47	212	+ 14
Sept.	73	− 15	1411	−135	1084	+ 18	203	−111	279	− 47
Oct.	108	+ 20	1816	−274	1328	−192	175	− 61	252	+ 20
Nov.	245	+ 59	2143	+207	900	−114	144	− 66	365	+ 1
Dec.	226	+ 24	1578	+ 46	712	−174	86	− 16	435	− 31
	1903.		1906.		1909.		1912.		1915.	
Jan.	141	− 45	1698	−120	811	−145	107	− 9	636	+ 8
Feb.	368	− 50	1560	− 50	852	− 60	78	− 40	549	+ 35
Mar.	301	− 83	1584	− 2	834	− 30	34	+ 14	727	− 57
April	470	−174	1765	− 33	525	− 17	13	− 9	725	− 83
May	540	+ 80	1690	+ 80	510	0	39	− 15	715	− 39
June	400	− 26	1858	−158	324	− 12	39	+ 11	795	+147
July	479	+209	2238	0	611	− 99	14	− 14	592	− 12
Aug.	613	−113	1370	−150	600	−102	12	+ 4	631	− 47
Sept.	381	− 39	968	− 72	386	−102	0	...	886	− 94
Oct.	348	− 66	750	−138	464	− 86	10	− 10	1012	+ 52
Nov.	251	+ 5	770	−104	487	+ 53	17	+ 1	1123	+ 19
Dec.	462	− 6	1339	− 43	506	− 36	52	− 4	866	− 80

TABLE III.

Mean Daily Areas of Faculæ (SOUTHERN *Hemisphere*), *with the Mean Differences in Area between the West and East Limbs, for each Month, from* 1886 *to* 1915.

Month.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.	Mean Daily Area.	W.−E.
	1886.		1889.		1892.		1895.		1898.	
Jan.	477	− 71	97	− 47	1190	−218	1162	− 74	295	+ 95
Feb.	515	− 31	52	− 4	1444	+ 16	1243	+ 69	764	+ 4
Mar.	712	−102	114	+ 14	1597	− 65	910	+204	790	− 66
April	567	− 11	13	− 5	1683	+373	1177	− 77	737	− 37
May	509	+ 75	71	+ 23	1949	− 89	1272	+ 16	767	+ 45
June	429	+ 43	125	+ 23	1387	+283	1178	− 64	564	− 98
July	273	+ 7	179	+ 33	1720	+ 10	1158	−134	299	− 13
Aug.	225	− 35	247	+119	1611	− 69	982	+140	512	+100
Sept.	339	− 7	170	− 42	1609	−209	806	− 2	742	+ 20
Oct.	223	− 55	74	+ 36	2235	−115	741	+ 51	744	−272
Nov.	264	+ 10	44	− 20	1934	+152	601	− 43	501	+ 71
Dec.	143	− 67	139	− 33	1810	+ 10	930	− 2	433	− 33
	1887.		1890.		1893.		1896.		1899.	
Jan.	194	+ 64	85	+ 35	1500	− 54	695	− 49	453	−129
Feb.	203	− 25	19	+ 17	1432	+ 92	588	−104	262	+ 12
Mar.	112	+ 22	20	+ 2	1209	+ 61	927	+115	388	− 6
April	77	− 15	22	+ 18	1548	− 98	650	+ 26	304	− 54
May	228	− 36	55	+ 25	1708	−216	554	− 6	232	− 4
June	226	− 32	83	+ 11	1692	+140	948	+146	301	− 19
July	252	− 8	258	− 72	1713	− 75	1052	− 16	306	− 2
Aug.	225	+ 7	109	+ 27	1991	+113	991	−205	187	−115
Sept.	222	− 38	222	+ 16	1630	− 78	1275	+ 81	101	− 9
Oct.	214	+ 72	441	+ 51	1382	−116	936	− 66	144	− 18
Nov.	152	− 48	333	− 47	821	− 31	962	+ 50	293	+ 23
Dec.	298	− 16	263	+ 45	935	−213	1127	−123	193	− 5
	1888.		1891.		1894.		1897.		1900.	
Jan.	256	+ 22	461	− 67	941	+ 27	1360	− 76	92	+ 26
Feb.	265	+ 31	301	− 17	1092	− 58	1055	− 1	96	+ 96
Mar.	271	− 53	216	− 56	757	+ 63	567	− 55	189	+ 11
April	230	− 56	277	− 1	1060	−246	670	+126	197	+ 21
May	224	+ 94	455	− 59	1211	+211	455	− 75	190	− 12
June	221	+ 53	302	+ 20	1078	+130	481	− 3	46	+ 4
July	172	− 38	601	− 1	1411	− 55	714	+122	133	− 39
Aug.	187	− 47	388	− 56	894	−164	915	−199	20	+ 2
Sept.	236	+ 14	347	+ 17	976	− 44	891	−163	67	+ 9
Oct.	95	− 11	532	− 40	1490	−102	445	+ 27	103	− 13
Nov.	170	− 58	524	− 48	1019	− 39	508	+ 56	63	+ 1
Dec.	107	+ 63	671	− 17	1126	+ 76	262	− 56	10	+ 6

TABLE III.—*continued.*

Mean Daily Areas of Faculæ (SOUTHERN *Hemisphere*), *with the mean Differences in Area between the West and East Limbs, for each Month, from* 1886 *to* 1915.

Month.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.	Mean Daily Area.	W.—E.
	1901.		1904.		1907.		1910.		1913.	
Jan.	0	...	542	+158	754	+ 54	621	− 73	85	+ 9
Feb.	25	− 25	378	− 70	845	− 15	693	− 57	42	0
Mar.	16	− 16	570	− 56	1252	−106	681	− 93	38	− 20
April	0	...	731	− 73	1092	− 72	480	− 40	47	+ 3
May	0	...	1052	−138	992	− 68	642	− 98	28	− 4
June	7	+ 7	666	− 34	942	− 46	675	− 59	37	+ 3
July	26	− 26	891	− 77	969	− 69	729	− 95	27	− 13
Aug.	0	...	997	− 11	903	− 33	607	− 23	5	− 5
Sept.	0	...	1077	+171	1228	− 24	799	−189	6	− 6
Oct.	34	− 22	882	−152	1154	− 40	871	+ 1	22	+ 10
Nov.	93	− 47	745	−117	1158	− 18	610	+ 16	30	+ 30
Dec.	2	− 2	808	+106	1312	−184	448	− 62	133	+ 20
	1902.		1905.		1908.		1911.		1914.	
Jan.	63	+ 11	879	− 89	1265	−211	442	−154	112	− 6
Feb.	12	− 8	1200	+ 16	1327	− 21	476	−120	51	+ 19
Mar.	13	+ 1	1034	− 48	1084	−106	511	− 85	83	− 31
April	0	...	936	− 50	1539	−283	522	− 94	123	+ 45
May	7	− 7	770	+ 96	1208	−154	507	− 5	86	− 46
June	18	+ 18	940	+ 68	1295	− 47	233	− 83	338	− 32
July	38	− 34	1251	− 65	1440	−184	209	− 53	351	− 1
Aug.	0	...	1394	−172	1397	+135	211	− 97	356	− 30
Sept.	102	− 84	1357	+ 65	1267	− 49	161	− 35	420	− 86
Oct.	199	+ 45	846	− 70	1514	−196	85	− 13	221	− 35
Nov.	156	+ 30	1002	−172	919	−195	153	− 17	261	−119
Dec.	90	− 48	1072	+ 46	892	−128	196	0	278	− 10
	1903.		1906.		1909.		1912.		1915.	
Jan.	79	− 37	1091	− 5	574	−158	112	− 40	296	− 6
Feb.	311	+ 47	739	−113	1075	−165	120	− 68	556	+ 20
Mar.	139	− 23	1006	−206	1165	−249	137	− 27	526	− 32
April	497	− 23	931	− 59	862	− 70	175	− 43	586	+ 38
May	430	− 70	731	− 53	829	−153	218	− 62	345	− 47
June	297	− 31	811	− 39	699	− 25	326	− 12	456	+ 60
July	894	−204	774	+100	714	−140	203	− 1	1215	− 27
Aug.	761	+ 87	1020	+ 76	580	− 6	90	− 24	1518	−132
Sept.	462	− 16	804	+ 58	715	+ 5	241	− 25	1116	− 68
Oct.	641	−141	669	+ 5	771	− 97	253	− 15	833	+ 23
Nov.	1141	− 25	791	+ 77	733	−169	116	− 40	831	+ 3
Dec.	1188	−208	858	− 18	660	− 42	118	− 4	688	− 40

TABLE IV.

Mean Daily Areas of Faculæ (ALL LATITUDES *included*).
For each Month from 1886 *to* 1915.

Year.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1886	818	669	906	715	810	625	462	458	595	317	347	256
7	307	345	332	202	367	323	325	326	289	271	243	316
8	337	298	354	254	237	284	212	189	259	100	225	131
9	100	58	144	64	71	130	197	253	174	116	67	186
1890	195	72	227	56	142	93	311	187	488	729	552	571
1	609	700	714	800	1217	1588	2375	1636	1688	1895	1840	1811
2	2778	3021	3211	3468	3771	3041	3205	3166	3165	4080	3239	3073
3	2408	2222	2146	2389	2742	2876	2778	3139	2473	2317	1430	1875
4	1504	1799	1340	1806	2140	1872	2527	1697	1805	2539	1624	1838
5	2197	2194	2117	2556	2584	2561	2235	2449	2346	2194	1847	2057
6	1399	1145	1495	1324	954	1212	1416	1255	2006	1500	1446	1773
7	1765	1802	1258	1401	972	656	889	1116	1408	815	679	1070
8	1125	1201	1040	913	955	755	434	757	948	1097	916	565
9	487	274	484	381	260	461	535	244	158	184	316	253
1900	183	376	250	318	418	126	178	29	119	103	72	20
1	...	25	30	...	63	44	47	6	3	34	93	7
2	91	14	186	131	180	200	56	38	175	307	401	316
3	220	679	440	967	970	697	1373	1374	843	989	1392	1650
4	1090	1034	1350	1630	2003	1620	2028	2110	1850	1970	1702	2716
5	2200	2648	2348	2329	2237	2368	2747	3256	2768	2662	3145	2650
6	2789	2299	2590	2696	2421	2669	3012	2390	1772	1419	1561	2197
7	2374	2395	2637	2089	1577	1413	1490	1478	2159	2111	2064	2238
8	2406	2467	1807	2006	1680	1832	2054	2336	2351	2842	1819	1604
9	1385	1927	1999	1387	1339	1023	1325	1180	1101	1235	1220	1166
1910	1047	1029	1053	748	1002	997	1118	855	1083	1180	876	662
1	578	577	644	695	625	356	382	468	364	260	297	282
2	219	198	171	188	257	365	217	102	241	263	133	170
3	222	127	122	79	52	55	79	26	61	64	76	176
4	178	108	198	463	444	664	400	568	699	473	626	713
5	932	1105	1253	1311	1060	1251	1807	2149	2002	1845	1954	1554
Means	1065	1094	1095	1112	1152	1072	1207	1175	1180	1197	1073	1130
(*a*)	1125	1138	1149	1231	1408	1339	1463	1350	1328	1456	1141	1211
(*b*)	856	930	888	939	901	814	970	1019	1028	966	1016	1102
(*c*)	1213	1213	1247	1166	1046	1063	1188	1155	1183	1169	1063	1076

their apparent areas are partly due to their extent over the solar surface, and to a small degree to their elevation; prominences can only be observed when actually on the Sun's limb, when their apparent areas are in the plane perpendicular to the Sun's surface. The method, therefore, adopted in Mrs. Maunder's paper of dividing the Sun's disc into 14 lunes, as well as into the two hemispheres north and south, could not be followed here. Each hemisphere yielded two quadrants, or four in all: N.W., N.E., S.W., S.E. But while in that paper the entire cycle, 1889–1913, was treated as a whole, in the present note the distribution of the faculæ between the east and west limbs is given for each month and for each year.

The chief relations brought out in the Tables and diagrams of the present paper are the following:—

1. Though the measurement of the areas of faculæ is attended with much greater difficulty and uncertainty than the measurement of sunspot areas, yet the mean daily areas month by month have a great consistency, and indicate that an accordant system of measurement has been established and maintained during the 30 years in question. Some difficulties attend the discussion of the observations of the 12 earliest years of the series, viz. 1874 to 1885, but I hope that it will prove possible to make some use of them in the future. But the most satisfactory way of increasing the material at our disposal will be by maintaining the same system of measurement as precisely as possible in the years to come.

2. Table IV. appears to show that there is no sufficient indication of an annual period in the areas of faculæ. There is a very slight excess in area for the six summer months, April to September, over the six winter months, when the 30 years are considered together. But the low value for June compared with the high value for July seems to show that this is accidental, not systematic. This conclusion is confirmed when the three decades are treated separately, as in the last three lines of the Table, marked (*a*), (*b*), and (*c*). In (*a*)—1886 to 1895—July and October were the most prolific months; in (*b*)—1896 to 1905—December; in (*c*)—1906 to 1915—January, February, and March.

3. The measured areas of the faculæ vary with the progress of the solar cycle in steady conformity with the variation in the areas of the spots. This holds good not merely for the spots and faculæ as a whole, but also for the relations of the spots and faculæ in the two hemispheres, north and south. As is well known, the progression of sunspot activity in the northern hemisphere differs as to the dates and amounts of its chief developments, from the same progression in the southern hemisphere. The progression of faculous activity in the north follows that of the northern spots in its chief characteristic features, though the two progressions differ in details. And similarly, the changes of the southern faculæ synchronise with and correspond to the changes of the southern spots. A comparison of the 6th and 10th columns in Table I.—viz. those headed "100 N.÷(N.+S.)"—will show how close this correspondence is.

4. The predominance in the numbers and areas of spot groups east of the central meridian of the Sun over those to the west of it, finds a parallel in the higher mean daily area of faculæ on the east limb than of those on the west limb. Taking the west limb as the standard, it supplies, on the average of the 30 years, only 485 per thousand of the faculous area as against the 515 of the east limb, a difference quite outside the limits of accidental error. For it must be borne in mind that the conditions of measurement of the areas of faculæ are exactly the same for all points of the solar circumference; the position angle of a facula from the N. end of the Sun's axis is not determined until the last stage of the reduction is reached.

This excess of 3 per cent. in faculous area of the eastern hemisphere over the western is the average for 30 years, and if future cycles of solar activity are comparable with these three past cycles, we should be disposed to expect that this proportion of excess should not be greatly altered with increase of material. It may be remembered that in Mrs. Maunder's paper (*ibid.*, p. 471) for the sunspots of 1889–1901, the excess in sunspot area of the eastern hemisphere over the western for the totality of areas of spots of all sizes and durations, amounted to 3 per cent. This excess is of the same order as that which I have now found for the excess in eastern faculæ, and as Mrs. Maunder found for eastern prominences, 1892–1905, viz. $2\frac{1}{2}$ per cent. For spots, she was able to find not only the excess for areas of all spots, but also for those of long duration in particular; for the excess of eastern spot groups observed consecutively for 13 or 14 days was 17 per cent.; for recurrent groups, 19 per cent.; for the excess of those coming into view at the east limb over those disappearing at the west limb was 22 per cent. I have not been able, from the nature of the case, to consider what effect would be seen if only long-lived faculæ were taken into account, nor the effect on the numbers of distinct groups of faculæ.

But Table I. indicates that the "Apparent Earth Effect" in the area of faculæ is of about the same order on the whole as the "Apparent Earth Effect" on the area of sunspots. The influence seems to be the same, acting in the same way and to a similar degree.

And this effect is not accidental in its nature; it is persistent and systematic. An eastern predominance of 3 per cent. may not seem an important one, but there is an aspect of it that cannot be expressed in percentages, though it is a striking one. Not only do the three cycles taken together show a predominance of east over west, but each of the separate cycles shows the same, on the whole, and in their northern and southern hemispheres separately. Thus, if we take the cycles year by year we find that in the 20 years from 1896 to 1915 in the northern hemisphere, in only 2 years did the total western faculæ exceed the eastern; in only 2 years did the same thing happen for the southern hemisphere; but in no single instance did the combined areas, north and south, of the western faculæ exceed those of the eastern. In the 10 years,

1886–1895, there were more frequent instances of the west exceeding the east, and the year 1916 was also one of western predominance.

5. The preponderance of the area of the faculæ appears to vary with the progress of the solar cycle. It is unfortunate that the 30 or 31 years here under consideration include only two complete cycles, if a cycle is always to be reckoned from the first signs of increasing activity. These two cycles comprise (*a*) the 12 years from 1890 to 1901, and (*b*) the 12 years from 1902 to 1913. Grouping together the 5 years when the solar activity was rising (*i.e.* 1890, 1891, 1902, 1903, and 1904), the 8 years of maximum (viz. 1892–1895 and 1905–1908), and the 11 remaining years of the two cycles, during which the activity was diminishing, it is clear that the eastern preponderance was very markedly greater when the solar activity was on the decline.

TABLE V.

Change in the Relative Areas of Faculæ, West and East, with the Progress of the Solar Cycle.

Phase.	No. of Years.	1000 W/(W+E).		
		North.	South.	All Latitudes.
Rise	5	502	476	491
Maximum	8	491	486	489
Fall	11	461	462	462
1890–1913	24	487	481	483
1886–1916	31	490	480	485

6. If instead of commencing a cycle, as it is most natural to do with the beginning of the rise of activity, we take any clearly marked phase of the solar curve and reckon the cycle as lasting till the next corresponding point is reached, then the figures at our disposal will enable us to present seven complete cycles, as shown in Table VI.

TABLE VI.

Indication of Long Period in Changes of the Relative Areas of Faculæ on the West and East Limbs.

Duration of Cycle.		No. of Years.	1000 W/(W+E)			Commencing Phase.
From.	To.		North.	South.	All.	
1886	1898	13	504	493	498	Middle point of fall.
1890	1901	12	503	492	497	Commencement of rise.
1892	1904	13	497	489	493	,, ,, maximum.
1896	1908	13	478	477	478	,, ,, fall.
1899	1910	12	474	467	473	Middle point of fall.
1902	1913	12	473	463	468	Commencement of rise.
1905	1916	12	479	468	474	,, ,, maximum.

Of these the second and sixth start with the commencement of the rise.

The Table brings a striking feature to light in that there is a continuous decline in the quantity 1000 W/(W + E) from the first of these cycles to the sixth; the seventh shows a slight recovery. It would appear, then, that from 1886 to 1913 there was a distinct

general tendency for the proportion of faculæ on the west limb to diminish, over and above their tendency to diminish during the fall of activity in any particular cycle. This suggestion of a long period variation is one of great interest, but a much longer series of observations will be required to establish it definitely.

The Tables and Diagrams illustrate the advantage of separating the spots and faculæ north of the Sun's equator from those south of it. The two hemispheres are closely connected in the varied phases of the solar cycle; the solar changes belong to the Sun as a whole. But the northern and southern hemispheres play quite distinguishable parts in those changes, and an important source of information is neglected if the statistics of the two regions are not given in such a manner that the features of each can be recognised.

Summary.

As in the cases of sunspots and of prominences, this predominance of area in faculæ on the eastern limb of the Sun, not only in total amount, but also in its steady persistency through many successive years, does not appear to be assignable purely to any solar cause as yet recognised. That which we term "east" on the Sun's apparent disc, and that which we term "west," are so termed solely from an earthly point of view. From a solar standpoint, that which we term the "east limb," "the central meridian," or "the west limb," are purely conventional and imaginary lines beneath which every portion of the Sun's surface moves in turn; they are not landmarks fixed by any solar detail. It would therefore appear in our present state of knowledge that the cause of the disproportion must be terrestrial, and terrestrial only. It cannot be due to any effect of foreshortening or arise from any asymmetrical effect due to the peculiar life history of the special phenomenon examined, such as a swift growth to maximum and a slow decline to minimum might be supposed to occasion. For when a very large number of cases of the phenomenon are considered, whether they be spots, faculæ, or prominences, any such asymmetrical effect must vanish. No particular phase in the life history of any of these should be associated with any special meridian of the Sun as seen from the Earth, unless the Earth was the cause of that phase. But sunspots, prominences, and faculæ are different in their solar levels, in their forms, in their structures, and in their development; therefore the common influence on them causing each to be larger or more numerous east of the central meridian of the Sun, cannot arise from any of these diverse characteristics.

In Mrs. Maunder's paper of 1907, the statistics given suggested that the "Apparent Earth Effect" was of the nature of a damping influence. The fact that the present inquiry seems to show that the eastern predominance is least marked during the increase of the solar activity and most marked during its decline, would suggest that the power of solar resistance to this damping was greatest when the general solar activity was increasing.

Cometary Radiant Points, 1875–1920. By the Rev. M. Davidson, B.A., M.Sc.

In the Report of the British Association, 1875, Prof. Herschel gavè a list of radiant points of comets up to the year 1874 which approached the orbit of the earth within one-fourth of an astronomical unit (pp. 232–5). I have examined the comets since that date, taking the orbit-elements from Dr. Galle's *Verzeichniss der Elemente der bisher berechneten Cometenbahnen*, *Chambers' List*, 1889–1908, the lists supplied in the *Journal of the British Astronomical Association*, *The Observatory*, and *M.N.R.A.S.*, and have computed the positions of the radiants of meteor showers from their dèbris. The greater number of these comets are assigned parabolic elements, but as many of them may be discovered in the future to be periodic and therefore capable of leaving cometary matter to produce meteor showers, it seems worth while including all, even those suspected of hyperbolic motion. The ellipticity of the orbits has been taken into consideration in the computation in Nos. 4, 22, 29, 33, 35, 36, 38, 39, 40 (29, 38, 40 refer to the same comet, 1881 v.); in the other cases, either the eccentricity is so great that the assumption of parabolic motion is sufficiently accurate, or the orbit is given as a parabola. The radiant point is given in longitude and latitude (λ, β), as well as in right ascension and declination (α, δ), in order that the positions of the radiant at dates prior or subsequent to the dates stated may be easily found. By increasing or decreasing the longitude 1° for each day after or before the date in column 3, leaving the latitude unchanged, the radiant point can be determined, and the coordinates λ, β transformed to α, δ, if necessary. In a few cases, on account of the very small inclination, the nearest approach is not at either node, and the letter N. or S. indicates if the comet was north or south of the orbit of the earth at the corresponding date. A number of radiant points have been computed for periodic comets which are given in Prof. Herschel's list, but in no case have the elements altered by a sufficient amount to make any practical difference in the position, and they are not given in the present list. I find, however, that Comet 1871 v. (No. 30) is not included in Prof. Herschel's catalogue; also that there is no mention of the near approach of Comet 568 ii. A.D., on September 18 (No. 34), the dates July 23 and August 5 only being given.

The eighth column gives the meteor speed, allowance being made for the increase due to the attraction of the earth. The last column contains the distance of the comet from the orbit of the earth in terms of an astronomical unit; as a rule no comets are included which pass more than 20 millions of miles from the earth's orbit; No. 42 exceeds this, however, by nearly 2 million miles.

In the *B.A.A. Journal*, 25, 293, I gave the position of the radiant for Giacobini as 267° + 50° on October 10. On looking through my figures I find that an error occurred by reading

$\omega = 191^\circ\ 28'$ instead of $171^\circ\ 28'$; the corrected position is given in No. 35. *Chambers' List* gives $q = 1{\cdot}2978$ for Comet 1895 ii., Swift, but in the *B.A.A. Journal*, **7**, 362, $\omega = 167^\circ\ 46'$, $\phi = 44^\circ\ 42'$, $T = 7{\cdot}2043$ years, and from these I find $\log q = 0{\cdot}0440$, and have adopted this value. Also, for Comet 1892 i., No. 15, some authorities give $\Omega = 246^\circ\ 54'$, and others $240^\circ\ 50'$; the latter has been used, but the difference does not make much alteration in the radiant point.

The present catalogue was sent to Mr. Denning for comparison with his list of meteor radiants, and he hopes to write something on the subject. Up to the present he has merely stated that there are some interesting agreements, but I am uncertain of the closeness of the correspondence.

No.	Comet and Node of Nearest Approach.	Date of Shower.	Radiant Point.				Meteor Speed. Miles per sec.	Comet's Distance from Orbit of Earth.
			λ.	β.	α.	δ.		
			°	°	°	°		
1	1888 iii., ☊	Jan. 3	146·7	−50·6	130·2	−34·7	29·1	0·192
2	Lowe, ☋	,, 25	177·4	23·2	187·3	22·2	37·3	0·002
3	Delavan (1913 *f*), ☊	,, 27	51·7	−39·4	59·4	−20	11·6	0·118
4	Denning ii. (1894 i.), N.	,, 31	40·6	−22·6	45·8	− 6·5	9·6	0·100
5	1887 iii., ☊	Feb. 5	212·7	−22·7	201·4	−33·6	42	0·130
6	1905 *a*, ☊	,, 27	66·3	−84·9	85·7	−61·8	18	0·210
7	1893 *c* (Brooks), ☊	Mar. 15	269·1	−28·4	268·7	−51·9	41	0·173
8	1877 i., ☊	,, 28	280·8	−15·3	283·3	−38·3	44	0·181
9	Schaumasse, ☋	,, 30	298·7	11·1	298·5	− 9·6	42·7	0·029
10	Reid, ☋	April 7	267·2	67·6	268·5	44·1	27·2	0·117
11	1885 iii., ☊	,, 14	235·9	−61	184·2	−74·1	25·3	0·133
12	1894 ii., ☊	,, 16	320·7	−52·9	356	−62	32	0·082
13	Quenisset, ☋	,, 24	331	37·4	319·5	23·8	36	0·023
14	1886 i., ☋	,, 26	343·5	50	322·5	39	30·7	0·209
15	Swift (1892 i.), ☊	May 21	225·6	−69·8	148·5	73·6	18·4	0·062
16	1885 v., ☊	June 13	261·9	−64·3	209·6	−86	19·7	0·174
17	Perrine, ☋	,, 16	360	20·2	351·7	18·3	43	0·171
18	1910 i., ☋	,, 20	313·8	9·4	313·6	− 7·7	36	0·135
19	Beljawsky, ☋	,, 20	45	27·5	32	42·3	33·7	0·131
20	1874 ii., ☊	,, 26	13·1	−18·2	19·2	−11·5	43·2	0·074
21	1898 x. (Brooks), ☋	,, 28	25·2	21·1	14·7	29·2	41	0·043
22	Pons-Winnecke, ☋	July 1	171·6	58·5	207·6	54·4	11 2	0·046
23	Morehouse, ☋	,, 5	16	23·8	4·6	28·1	42·5	0·067
24	1886 iii., ☊	,, 10	357·2	−45	19·5	−41·5	35	0·070
25	1875 i., ☋	,, 14	175·6	32·8	190·6	31·5	11·6	0·174
26	1902 *d* (Giacobini), ☋	,, 15	90	19·2	90	42·6	27·5	0·220
27	Daniel, ☊	,, 28	12·3	−82·7	73·6	−64	22	0·172

No.	Comet and Node of Nearest Approach.	Date of Shower.	Radiant Point.				Meteor Speed. Miles per sec.	Comet's Distance from Orbit of Earth.
			λ.	β.	α.	δ.		
28	1883 i., ♂	July 30	83°·9	45°·7	78°·1	68°·9	30	0·105
29	1881 v., S.	Aug. 4	303·2	10	303·2	− 9·7	17·7	0·108
30	1871 v., ♂	,, 20	25·2	40·7	3	46·7	34·5	0·062
31	1900 ii., ☊	,, 21	36·7	−73·3	66·8	−54·1	25	0·015
32	Rordame, ☊	,, 31	53·4	−11	53·8	8	43·5	0·206
33	1895 ii. (Swift), ♂	Sept. 13	241	11·7	241·8	− 9	9·6	0·114
34	568 ii., A.D., N.	,, 18	211·6	9·6	212·7	− 3·1	12·8	0·043
35	Giacobini, ♂	Oct. 9	221·8	76·2	251·5	55·9	14·5	0·060
36	(Tempel-Swift), 1891 v., N.	Nov. 6	314	23	310	5·5	9·2	0·138
37	1879 ii., ☊	,, 8	136	−43·8	125·2	−25·8	36·7	0·117
38	1881 v., ☊	,, 28	265·7	−13·8	264·8	−36·8	15	0·131
39	1896 *g* (Perrine), ♂	,, 29	307·9	46·3	298·2	26·2	11	0·149
40	1881 v., N.	Dec. 15	271·9	−10·5	272	−34	17·7	0·069
41	1895 iii. (Brooks), ☊	,, 15	218·7	−49·3	188	−59·1	29·5	0·154
42	1906 *g*, ☊	,, 16	149·6	−78·4	109·7	−59·1	23	0·235
43	1883 i., ♂	,, 30	234·5	45·8	243·3	25·6	30·3	0·137

Preliminary Orbits of Σ2052 *and* OΣ327.

By J. Jackson, M.A., B.Sc.

(*Communicated by the Astronomer Royal.*)

The notation used is the same as that in *M.N.*, **80**, 543.

7642. Σ2052. B.D. 18°, 3182.

R.A. $16^h\ 25^m\ 36^s$. Mags. 7·5, 7·5. P.M. 0″·508 in 320°·1 (Porter).
Dec. +18° 34′. Spec. G5. π + 0″·08 ± ″·044 (Chase).

This is a binary which has recently shown rapid orbital motion. Between 1830 and 1910 the position-angle decreased by 23°, while the distance decreased from 3″ to 1″. Both continued to decrease, and the observations of 1917 showed a further decrease of about 40° in position-angle, while the distance was measured as 0″·27. Further rapid changes were to be expected. Two observations at Greenwich in 1919 showed a further reduction in position-angle of 17°, but to get a constant rate of description of area this had to be altered by 180°. As the two components are stated to be

equal this is permissible, and in fact it is the only possible interpretation of the 1917 and 1919 observations, provided they are substantially correct except as to this uncertainty. The ephemeris given below shows that at the present time this is a very close pair. It should therefore be very carefully observed for the next few years. By 1930 the separation should again reach 1″.

The elements are only preliminary, and it should be possible to improve them as soon as new observations are available. No attempt has been made to fit the observations of 1917 and 1919

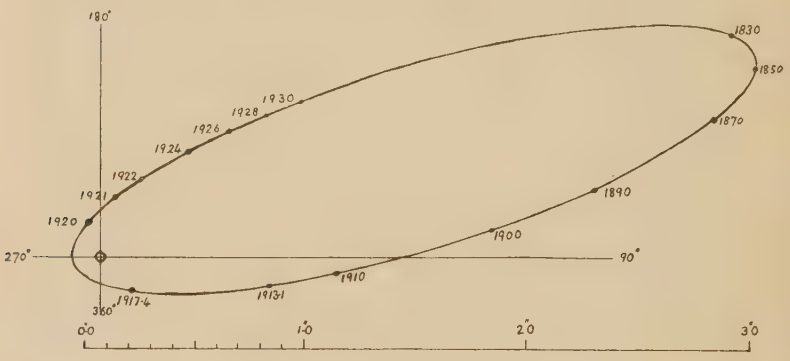


FIG. 1.—Orbit of Σ2052.

accurately. This could be easily done by a slight increase in the eccentricity—fixed at the round figure 0·92—and a slight change of epoch. The elements have been deduced by Zwiers' method.

T	1918·81	a	1″·62	i	50°·0
P	153·2 years	e	0·92	Ω	101·1
n	−2°·35			ω	189 6.

The ephemeris based on these elements is

1830	109°·4	3″·015	1921·0	165°·8	0″·287
1850	106·0	3·074	1922·0	152·5	0·404
1870	102·5	2·839	1923·0	145·1	0·518
1890	97·8	2·232	1924·0	140·4	0·624
1900	94·0	1·770	1925·0	136·9	0·724
1910	86·0	1·068	1926·0	134·3	0·818
1913·1	80·2	0·768	1927·0	132·2	0·908
1917·44	41·7	0·211	1928·0	130·5	0·993
1919·60	220·1	0·148	1929·0	129·0	1·074
1920·00	196·7	0·177	1930·0	127·8	1·151

The following is a comparison of the observed and computed positions :—

	Obs.	Comp.	O – C.	Obs.	Comp.	O – C.	
1829·52	109°·7	109°·4	+ 0°·3	2″·98	3″·01	– ″·03	Σ 3
1842·44	109·8	107·3	+ 2·5	2·40	3·07	– ·67	Ma 1
1851·89	108·0	105·7	+ 2·3	3·05	3·06	– ·01	Σ 1
1856·49	104·1	104·9	– 0·8	2·95	3·02	– ·07	Se 2
1858·44	104·8	104·6	+ 0·2	2·62	2·99	– ·37	Wr 3
1865·55	103·2	103·4	– 0·2	2·99	2·92	+ ·07	Se 3
1866·49	103·1	103·2	– 0·1	2·75	2·90	– ·15	Δ 4
1876·31	101·0	101·2	– 0·2	2·61	2·69	– ·08	Sp 5
1879·56	101·6	100·4	+ 1·2	2·70	2·61	+ ·09	Hl 3
1884·48	99·2	99·3	– 0·1	2·47	2·46	+ ·01	Hl 3
1890·42	98·5	97·7	+ 0·8	2·29	2·24	+ ·05	Gla 2
1897·91	94·7	95·0	– 0·3	1·89	1·90	– ·01	Gr. Obs. 10
1903·39	92·0	91·6	+ 0·4	1·56	1·57	– ·01	Gr. Obs. 19
1903·40	91·4	91·6	– 0·2	1·55	1·57	– ·02	Do 3
1908·56	88·7	87·6	+ 1·1	1·43	1·20	+ ·23	J 3
1909·38	86·4	86·6	– 0·2	1·09	1·13	– ·04	A 1
1909·41	86·2	86·6	– 0·4	1·08	1·13	– ·05	J 3
1909·45	84·1	86·6	– 2·5	1·37	1·13	+ ·24	Doo 3
1910·46	84·4	85·4	– 1·0	0·98	1·03	– ·05	Fox 3
1911·42	84·1	83·9	+ 0·2	1·04	0·95	+ ·09	A 1
1911·43	83·1	83·9	– 0·8	1·10	0·95	+ ·15	Voute 6
1912·45	80·9	81·8	– 0·9	1·00	0·85	+ ·15	Voute 4
1913·08	79·8	80·2	– 0·4	0·78	0·77	+ ·01	Gr. Obs. 11
1917·44	47·0	41·7	+ 5·3	0·27	0·21	+ ·06	J 2
1919·60	210·1	220·1	– 10·0	0·31	0·15	+ ·16	Gr. Obs. 2

The hypothetical parallax is ″·0449.

7936. OΣ327. B.D. 56°, 1959.

R.A. $17^h\ 12^m\ 43^s$. Mags. 7·6, 7·9. P.M.
Dec. +56° 13′. Spec.

The orbit of this binary appears to be of the same kind as that of OΣ185 described in *M.N.*, 80, 586. Modern observations with large refractors show a definite increase in the position-angle and in the separation, as is indicated by the positions :—

1891	301°·3	0″·22
1898	313·6	0·32
1905	320·6	0·38
1912	326·0	0·42.

With the aid of the curve joining these points, we can proceed to investigate the earlier observations. It would clearly be a

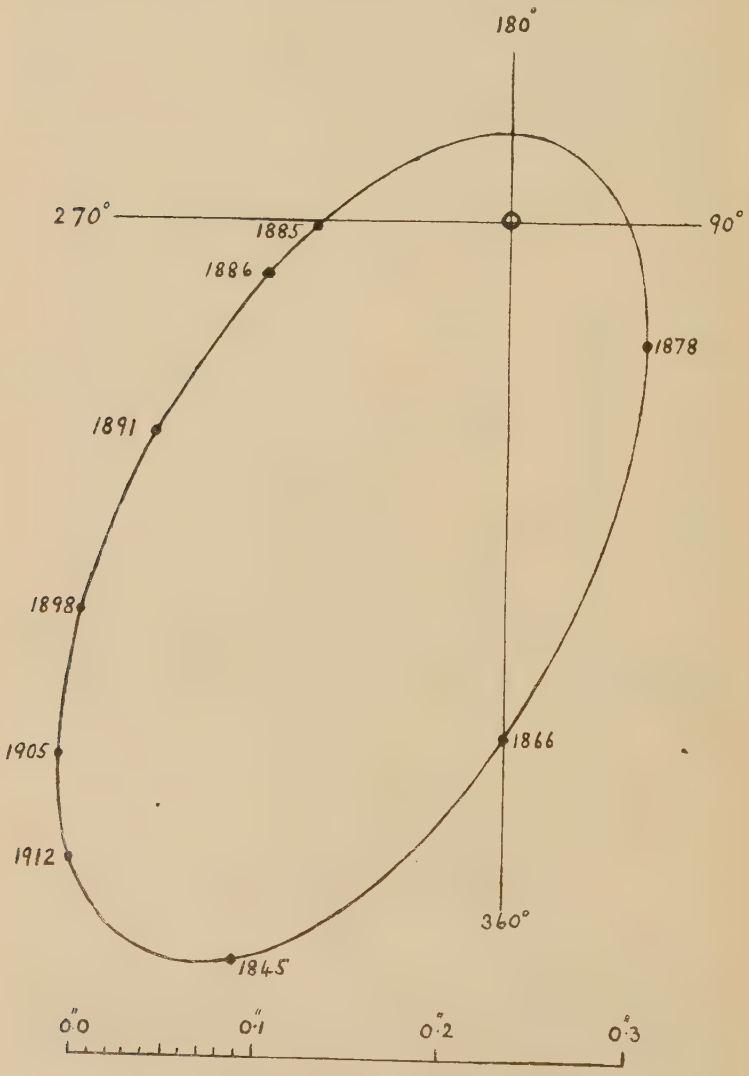


FIG. 2.—Probable Orbit of OΣ327.

difficult pair before 1890. First we find a doubtful elongation in the direction 90° observed by Perrotin with the 15-inch refractor of Nice in 1886. As most of the observations—even modern ones made with the largest instruments—appear doubtful

as to quadrant, this can be read as 270°. In the orbit shown in the diagram we find the position at that time 282°, 0″·14, which sufficiently explains the observation. In 1885, when Perrotin failed to separate the components, and in 1878, when Burnham failed, the separation would be about 0″·10. Proceeding to still earlier observations, we find that it was too difficult for Dembowski in 1866. For the orbit adopted here, the separation at that date was 0″·28 in position-angle 0°. It would certainly be a difficult object for Dembowski's 7-inch telescope, but it is not improbable that that exceptional observer would have noted an elongation had it been so large as 0″·28. The orbit might be altered to give a smaller separation at that date by increasing the separation between 1875 and 1885, but it has not been considered that the orbit can be so accurately investigated.

Finally, we come to the positive observations of Otto Struve and Maedler, who each observed it on four nights between 1842 and 1850. The means of their position-angles are

OΣ	1846·45	340°·6
Ma	1844·37	159°·7.

It is seen that these two observers differ as to the quadrant, and the question is, which of the two are in agreement with that adopted for the modern observations? We have decided to adopt the quadrant given by Otto Struve, for two reasons. (1) He had the larger telescope; (2) the adoption of this quadrant allows of the pair remaining really difficult for a longer time. If Maedler's quadrant were correct, the pair could not have been so close as 0″·1 at the times of the negative observations by Burnham and Perrotin, and the separation at the time of Dembowski's observation would not have been less. We thus come to the conclusion that the companion was in 1912 within 14° of the position in which it was in 1845. However, as the companion is near apastron, in that position it will take 20 years to complete the revolution. The period may therefore be about 88 years, although there is a possibility that it is twice as long.

It has not been considered worth the labour of computing the elements from the orbit which has been drawn. It may be said, however, that the eccentricity of the true orbit is near 0·80, and that the semi-major axis is about 0″·26. This pair should be carefully observed to determine whether the angle is increasing, as predicted. The positions should be roughly:—

1920	331°·3	0″·44
1930	337°·8	0″·44.

The above figures give a hypothetical parallax of ″·0104.

Photometric Measures of the Nuclei of some Typical Spiral Nebulæ. By J. H. Reynolds.

In any discussion as to the nature of spiral nebulæ the character of the nuclei of individual nebulæ is obviously very important.

In 1913 the writer made some photometric measures of the nucleus of the Andromeda Nebula, and showed that the light distribution along the major axis of the ellipse symmetrically to the centre point of the nucleus was a close approximation to the curve $(x+1)^2y = \text{constant}$. This result has been employed by Knut Lundmark * in a comparison with Schwarzschild's formula for stellar density in our own stellar system, and he concludes that there is a much greater condensation of stars in the Andromeda Nebula (if it is to be regarded as a remote stellar galaxy) than can be inferred in our own system according to Schwarzschild. The nucleus of the Andromeda Nebula is, however, only an example of a certain type, and is very dissimilar to the nuclei of many of the spiral nebulæ. It is possible to divide these into several classes depending on the type of nebulosity composing them.

Broadly speaking, there are two main types of nebulosity in photographs of spirals, the amorphous cloudy type and the condensed granular type, and an examination of many photographs of spiral nebulæ, published and unpublished, makes it evident that most of the examples can be classed in a series representing various stages of nebular condensation.

The following is a classification made up on these lines, the so-called "spindle" nebulæ, which are undoubtedly spirals seen edgewise, not being included:—

I. Spirals consisting entirely of amorphous nebulosity: examples of this are not numerous, but *N.G.C.* 205 (one of the two nebulæ near the Andromeda Nebula) and *N.G.C.* 3623 may be cited.

II. Spirals showing incipient condensation in the outer whorls only, the greater part of the nebula consisting of amorphous nebulosity. The Andromeda Nebula itself is a good example of this, the outer regions being broken up into comparatively bright and disconnected nodules on a hazy background.

III. In this class the condensation in the outer regions has advanced considerably towards the nucleus. This is the stage reached by *N.G.C.* 3031 (Ursa Major), where the outer half is of the condensed type, including what appear to be actually star discs, while the nuclear region is of the amorphous type and has a light distribution similar to the Andromeda Nebula. Another example is *N.G.C.* 2841 (Ursa Major).

IV. Includes the great majority of spirals. The nebular condensations appear in all regions except the nucleus itself, but a hazy background is still more or less conspicuous.

V. An advanced stage of Class IV.: the whole nebula is of the condensed type, the nucleus often having a definite boundary

* *Kungl. Svenska vetenskapsakademiens Handlingar*, Band 60, No. 8.

like a planetary nebula. The hazy background has practically disappeared in all regions. Example *N.G.C.* 5457 (Ursa Major. M. 101).

VI. In certain of the spirals another mode of condensation seems to be taking place. In these examples the nebular condensations appear in the intermediate regions between the nucleus and the outer whorls, usually along an area of which the nuclear point is the centre. These condensations are themselves the starting-points of numbers of spiral arms consisting of faint amorphous nebulosity which extends to a considerable distance. A good example of this is *N.G.C.* 4736 (Canum Ven. M. 94).

VII. There are others which do not seem to have followed symmetrical lines of development at all. Such are spirals of the coarse granular type which have apparently no definite nucleus at all, such as *N.G.C.* 253 (Sculptor), and spirals of irregular brightness and form like *N.G.C.* 4900.

A list of objects representing each of these classes has been made out, and exposures made on as many of these as was found practicable.

In this paper the light distribution in five spiral nebulæ along a single diameter has been determined. Short exposures were given so that the nucleus was not blotted out by over-exposure, and the photographic light values thus destroyed; consequently, all the photographs are under-exposed for the outer regions, which are usually much fainter than the nucleus itself.

A serious difficulty in this photometric work on nebulæ is that there is no standard of magnitude of nebular light, nor does it seem possible to devise one which would be of universal application. A certain rough standard may be devised for a particular instrument based on density and exposure times, but this would not be applicable to an instrument of different focal ratio. The out-of-focus image of a star has been suggested as a standard, but this is not suitable to a reflector, although it is a possibility with a refractor. At the same time, if a scale of photographic densities corresponding to known light values be adopted, it is evident that the measures of the photographic image of a nebula would be consistent amongst themselves, especially if two or more negatives of varying exposures were measured and compared.

In the following measures, therefore, no attempt has been made to bring the various examples to the same scale of nebular magnitude, but the light values given for an individual nebula may be regarded as correct within a probable range of 10 per cent. to 20 per cent. for high and low values, and of 5 per cent. to 10 per cent. for medium values, the ratio of density to exposure time being greater for the medium densities. If duplicate measures are made it is rare that the results obtained are the same in all respects, and there is always the difficulty of centering the nucleus. The grain of the plate also leads to anomalous results, but the general type and height of curve is not affected by any of these considerations

to an appreciable extent. The methods of obtaining the light values corresponding to a certain density of the film are dealt with in *M.N.*, **80**, 2.

It is necessary to emphasise the range of spectrum covered by these photographs, as with one exception unbathed plates were used which are insensitive to the visual spectrum in the longer wave-lengths. It has been shown at Mount Wilson and Upsala that the nuclei of the spiral nebulæ are comparatively brighter in the green region than the outer whorls when compared with photographs taken with unbathed plates. This means that the photometric curve would be still steeper if a green-sensitive plate were used. Further work is necessary to deal with the question of colour distribution in spirals, and it would be very desirable to make comparative photometric measures of individual nebulæ with unscreened panchromatic plates and unbathed plates.

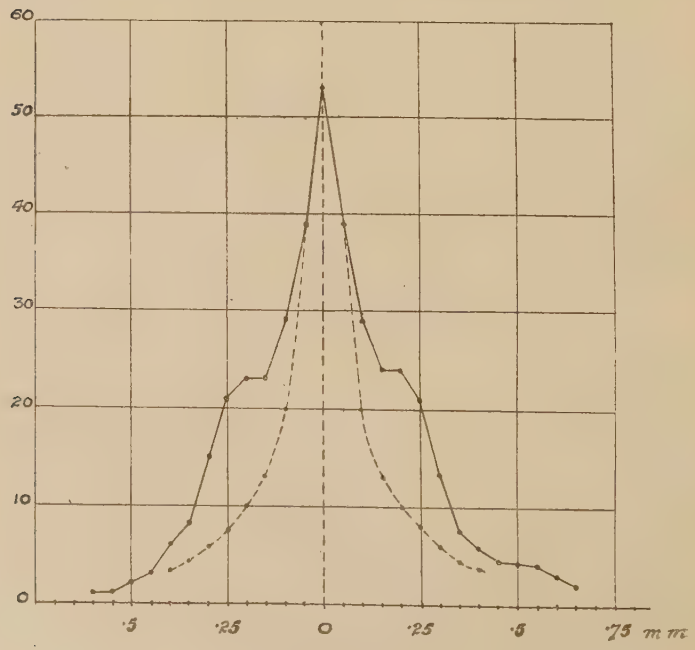
Class I. *N.G.C.* 3623. (*M* 65 *Leonis.*)

$\alpha = 11^h\ 13^m\ 43^s$ $\quad \delta = +13^\circ\ 38'{\cdot}5$

$\lambda = 209^\circ$ $\quad \beta = +65^\circ$

Exposure 81 m., 30-in. Helwân reflector, 1913 Feb. Scale 1 mm. = 59″.

This is a much elongated spiral with apparently no nebular



N.G.C. 3623.

condensations, although two faint objects on the outer whorl to the east of the nebula may be either of this class or stars.

The curve shown is much flatter than that obtained for the Andromeda Nebula. The curve $xy = \text{constant}$ is given for comparison.

Class IV. *N.G.C.* 4254. (*M* 99 *Virginis.*)

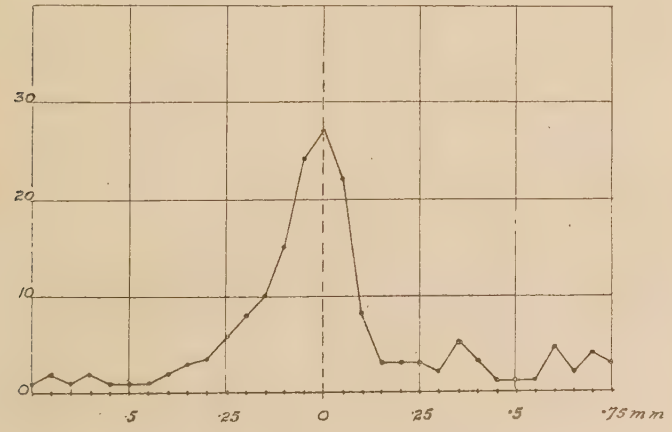
$\alpha = 12^h\ 11^m\ 43^s$ $\quad \delta = +15^\circ\ 12'$

$\lambda = 243^\circ$ $\quad \beta = +74^\circ$

Exposure 81 m., 30-in. Helwân reflector, 1913 Feb. Scale 1 mm. = 59″.

This is an outlying member of the great concentration of spirals in Virgo, and is correctly described by Rosse and Lassell as a three-branched spiral. Two of the branches are formed in the normal way on opposite sides of the nucleus, the third having its origin as a bifurcation of one of the normal spiral arms.

The nuclear region is free from nebular condensations, which are numerous in the outer regions.



N.G.C. 4254.

The light distribution of the nebula is more regular on one side than the other, but the curve $(x+1)^2y = \text{constant}$ is more consistent with the light distribution than any other curve of this order.

Class V. *N.G.C.* 4321. (*M* 100 *Comæ.*)

$\alpha = 12^h\ 15^m\ 26^s$ $\quad \delta = +16^\circ\ 36'$

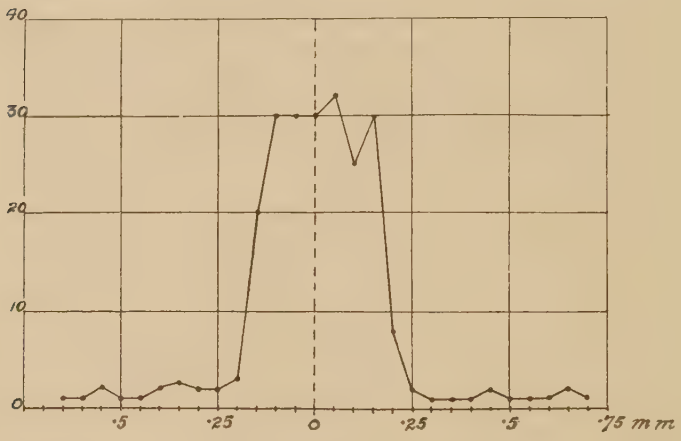
$\lambda = 249^\circ$ $\quad \beta = +77^\circ$

Exposure 81 m., 30-in. Helwân reflector, 1913.

„ 35 m. „ „

Scale 1 mm. = 59″.

The nucleus is a small well-defined ellipse of almost even luminosity: it is cut into three portions, presumably by filaments of absorbing matter, and is surrounded by a faint haziness extending for some considerable distance. The spiral arms and condensations are also faint.



N.G.C. 4321.

This curve is one that is especially difficult to reconcile with any theory yet put forward as to the nature of the spiral nebulæ. It, however, is not a usual type as far as this investigation has gone: another example of this type is *N.G.C.* 5383.

Class VI. *N.G.C.* 4736. (*M* 94 *Canum Ven.*)

$\alpha = 12^h\ 46^m\ 59^s$ $\quad \delta = +41^\circ\ 34'$

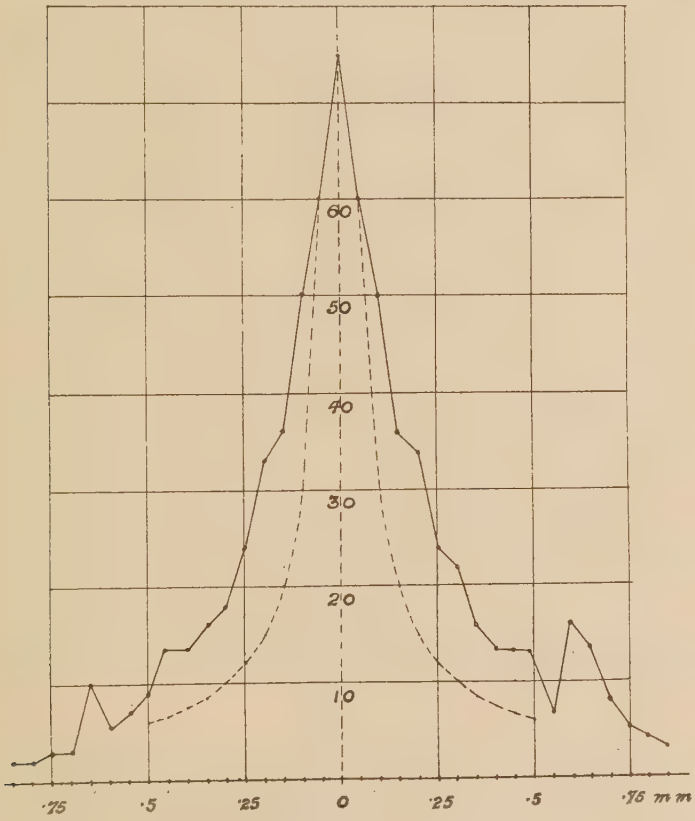
$\lambda = 85^\circ$ $\quad \beta = +76^\circ$

Three exposures, 28-in. Harborne reflector. Scale 1 mm. = 67″·7.

A fine series of graduated exposures of this nebula is given in *Mount Wilson Cont.*, No. 132.

The great comparative brightness of the nucleus made it difficult to obtain a reliable series of measures of the central region, and the ordinate of the nuclear point is doubtful.

The nuclear region is entirely amorphous. The ring of nebular condensations is shown by the increase of brightness between ·5



N.G.C. 4736.

and ·75 on either side, but the point of measurement on the left side fell on an interval between two of the condensations.

The curve xy = constant is given for comparison.

Class VI. *N.G.C.* 4826. (*M* 64 *Comæ.*)

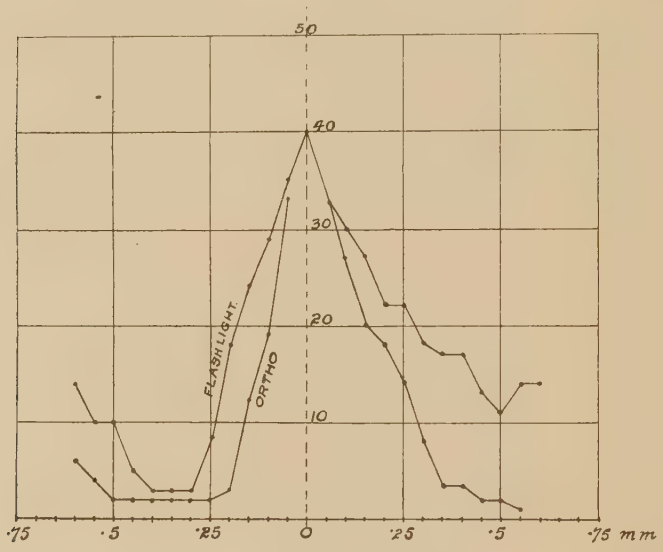
$\alpha = 12^h\ 51^m\ 49^s$ $\quad \delta = +22^\circ\ 13'$

$\lambda = 285^\circ$ $\quad \beta = +81^\circ$

Three exposures, 28-in. Harborne reflector, 1920 April. Scale 1 mm. = 67″·7.

It is doubtful whether this well-known nebula should not have been included in Class I., as it consists almost entirely of amorphous nebulosity, the only condensations occurring along the dark rift

near the nucleus. Two exposures were made on Imperial Flash-light plates and one on Wellington Antiscreen (unscreened). The curves measured from both kinds of plates are given.



N.G.C. 4826.

The comparative curves establish the difference in steepness of curve between the green and blue rays, which was to be expected from Seares' work at Mount Wilson.

Conclusions.

Although further examples are desirable, the results of these measures are at any rate sufficient for some conclusions to be drawn. It seems difficult to reconcile the various stages of condensation with the idea that the spirals are all distant galaxies. That they may ultimately become galaxies is a possibility, though even here the number of nebular condensations (which may be, and probably are, stars in process of formation) is in most examples very small compared with the numbers of stars in the globular clusters. The nuclear regions of the spirals consisting of amorphous nebulosity give a fairly symmetrical curve, but appear to vary very much in steepness. On the other hand, condensed nebulæ of the type of *N.G.C.* 4321 present characteristics to which we seem to have no clue whatever. It is also difficult to explain the undoubted fact that the outer regions of the spirals are bluer than the nucleus: some exposures made on the Andromeda Nebula by the writer suggest that this effect is still more pronounced in the red region of the spectrum than in the green. The theory of reflection from stars involved in the nebulosity does

not receive much support from these measures either, as the curves are as a rule flatter than that found for the Andromeda Nebula.

The novæ which have been recently discovered in various spirals have not as yet yielded any spectroscopic results which would enable us to class them with the galactic novæ, and the great example of this class which appeared in the Andromeda Nebula in 1885 and attained the seventh magnitude, unfortunately occurred before the spectroscopic investigation of the fainter stars was undertaken by photography, but the contemporary visual spectroscopic observations of this nova failed to show the characteristic hydrogen radiations which are typical of galactic novæ. It seems that too little attention has been paid to this object, which, if we are to regard the Andromeda Nebula as a galaxy at a distance of only twenty-five galactic diameters, must be credited with an intrinsic brilliancy far exceeding any of the galactic novæ. (Weiss, *Schriften Wiener Vereins*, Band v.)

There is no observational reason why the amorphous nebulosity should be considered as actually a conglomeration of star discs; in fact, all the photographic evidence is against it. Such objects, indeed, as *N.G.C.* 4286 have every appearance of being formless clouds of dust and haze. (*Cf.* J. H. Jeans, *Problems of Cosmogony*, p. 215.)

It seems safest to adopt for the present an agnostic position as to the real nature of the spiral nebulæ; there always is the possibility that they represent formations for which there are no analogies in the galactic system at all.

The absorption lines in their spectra may be produced by comparatively cool gases composing the amorphous nebulosity, the hotter nucleus and nebular condensations radiating a continuous spectrum.

Note on some Occultations of Stars by Planets.

By Arthur Burnet.

(*Communicated by Professor H. H. Turner.*)

The occultation of the star 4649, Bordeaux Catalogue (1890), by Venus, which was predicted to occur 1920 January 4,* was observed at some stations in America. Owing, however, to the faintness of the star (eighth magnitude) and unfavourable atmospheric conditions, the observations were not altogether successful.

Professor Bailey transmits the following account of the occultation as seen by Mr. H. Eaton, a student at Harvard Observatory:—

"Owing to the great difference in magnitude an accurate observation of immersion (on the bright limb) was impossible, the star disappearing from view about $23^h\ 19^m$ G.M.T. Emersion was observed at $23^h\ 30^m\ 47^s$ G.M.T. I should estimate the probable error of the observation of emersion as about -5^s. The

* *M.N.*, **80**, 92, and *Harvard Bulletin*, 703.

star was certainly seen at the time indicated. Telescope used 12-inch refractor; magnification about 90×."

Professor Mitchell of the Leander M'Cormick Observatory has communicated to Professor Turner the following observation made there by Dr. C. P. Olivier:—

"At the time of the occultation by Venus on January 4 the sky was clear, but the seeing was poor. On account of the low altitude, the glare from the planet, and the poor seeing, the conditions were very bad, although Venus was placed behind an occulting disk to cut down the glare. The star was followed until $23^h\ 13^m$, when at a distance of one and a half diameters of the planet it disappeared in the glare of Venus. The occultation occurred probably near the computed time."

Professor W. H. Pickering, writing from Mandeville, Jamaica, says:—

"The primary facts relating to the observation are as follows:—Our longitude is $5^h\ 10^m\ 2^s$ west, latitude 18° 1′ north, aperture of objective 11 inches, magnification employed 430. Computed time of disappearance $18^h\ 23^m$ E.S.T. Estimated altitude of Venus 43°. Character of the seeing on the standard scale 7. The sky was perfectly transparent and cloudless.

"At $18^h\ 8^m$ dawn was bright, the star was easily seen and about 30″ from the limb of the planet. The following measures were secured. The last was difficult and was only an occasional glimpse. Distance from limb about 8″.

E.S.T.	N. Cusp.	S. Cusp.
$18^h\ 06^m$	110°·9	
18 08	...	90°·0
18 13	...	75°·7
18 15	127°·5	

"The star was finally lost in the glare of light around the planet, due to reflection from the lenses rather than from the brightness of dawn.

"The star was looked for from $18^h\ 26^m$ to $18^h\ 32^m$, but could not be seen.

"Estimated distance from limb:—

$$18^h\ 08^m = 30'' \qquad 18^h\ 15^m = 8''.$$

"The planet was moving 3″ per minute, so that the estimates corroborate each other, but the occultation must have occurred earlier than expected."

At the Washington Naval Observatory, poor seeing prevented any observation. From the foregoing observations it is evident that, for Venus, the limit of magnitude of the occulted stars cannot as a rule be much fainter than 7 in order to secure successful observations. It may, however, be of interest to mention the case of the occultation of B.D. +2°, 2422 by Venus, which was observed at the Hector Observatory, Wellington, by Dr. C. E. Adams, 1919 August 1. The magnitude of this star is only 8·7.

"With the 5½-inch refractor, the disappearance was observed at $18^h\ 41^m\ 41^s$. The star then reappeared momentarily and disappeared again at $18^h\ 41^m\ 44^s$. The watch correction was $17^s{\cdot}76$. This gives the G.M.T. of disappearance as $18^h\ 42^m\ 0^s{\cdot}26$. There was no chance to observe the reappearance, as the sky clouded up."

An explanation of the difference between these two observations may probably be found in the fact that the New Zealand observation was a contact with the dark limb, and also that the fraction of the disc illuminated was only 0·30 as against 0·69 on 1920 January 4.

Occultation of a Star by Mars.

The eighth magnitude star B.D. -21°, 4230 will be occulted by Mars 1920 September 5. The catalogue places of the star, reduced to 1900·0, are as follows:—

Yarnall (Washington) 1860. Mean date of observations: R.A. 1864·0, Dec. 1870.0.

R.A. $15^h\ 50^m\ 57^s{\cdot}59$ Dec. $-21^\circ\ 55'\ 38''{\cdot}4$.

Algiers (1900). Mean date: R.A. and Dec. 1890.

$15^h\ 50^m\ 57^s{\cdot}57$ $-21^\circ\ 55'\ 39''{\cdot}2$.

I am indebted to H.M. Astronomer at the Cape for a plate taken 1920 March 27, from which I have derived the following position, reduced to 1900·0:—

$15^h\ 50^m\ 57^s{\cdot}56$ $-21^\circ\ 55'\ 41''{\cdot}0$.

The three places consistently indicate an annual P.M. of $-{}^s{\cdot}0006$ in R.A. and $-0''{\cdot}05$ in declination.

Geocentric conjunction occurs 1920 September 5, the apparent positions being:—

R.A. of planet and star	$15^h\ 52^m\ 11^s{\cdot}13$
Declination of Mars	$-21^\circ\ 59'\ 16''{\cdot}5$
,, star	$-21^\circ\ 59'\ 17''{\cdot}8$.

The occultation as seen from the Cape or Johannesburg will last from $6^h\ 52^m$ to $6^h\ 56^m$. At Algiers the planet will be at a fair altitude and the phenomenon will occur about 40 minutes after sunset, but the effect of parallax will be to place the planet well to the south of the star, and the occultation, if it takes place at all, will be very brief.

Ross' corrections have in this computation been applied to the places of Mars.

University Observatory, Oxford:
1920 *June* 11.

[*Note added June* 19.—As the result of further examination of the plate referred to above, I find that the star B.D. -21°, 4243

will also be occulted by Mars. The star is rather faint (mag. 9·1), but any observations would be useful as an indication of the limit of visibility in the case of an occultation by Mars. Professor E. S. Holden records the occultation of an 11th magnitude star by Mars, observed at Lick 1888 July 3 (*A.J.*, **181**).

The position of B.D. −21° 4243 is as follows:—

		h	m	s		°	′	″
1920·0	R.A.	15	54	46·09	Dec.	−22	7	10·5
1920 Sept. 6		15	54	49·08		22	7	14·0.

Geocentric conjunction in R.A. occurs 1920 September 6, $5^h\ 56^m$ G.M.T., the declination of the planet being $-22°\ 7'\ 18''·0$.

At the Cape there will be an occultation of two or three minutes' duration at $6^h\ 17^m$ G.M.T., but for Johannesburg an occultation is doubtful, though there may be a graze on the southern limb of the planet.

Observation of an Occultation of a Seventh Magnitude Star by Saturn 1920 *March* 14. By T. P. Bhaskaran, M.A.

The circumstances of the occultation were predicted by Mr Arthur Burnet and given in his paper in the *Monthly Notices of the R.A.S.*, **80**, 91. The following observations of the phenomenon were made with the 10-inch following telescope of the Hyderabad Astrographic Equatorial.

The star occulted was No. 1460 Cape (1900), magnitude 7·3—the same as A.G.C. Leipzig I. 4091, magnitude 7·6—and the predicted time of disappearance behind the ring was $0^h\ 5^m$ Indian Standard Time.

At $23^h\ 50^m$ the instrument was pointed to Saturn, and both the star and planet were seen together in the field. The star was a little north of the plane of the rings and preceding the ball; the distance from the disc of the planet was estimated to be about 6″.

From about 0^h midnight a continuous watch was kept, but it was not till $0^h\ 20^m$ that the star appeared to touch the ring very near the place where the ring itself apparently intersected the ball of the planet. At $0^h\ 22^m$ the star was observed to be behind the ring; it was gradually losing brightness, and the colour which was originally reddish turned pale white. The time of complete disappearance behind the planet was

$$0^h\ 23^m\ 19^s·2.$$

The emersion of the star from the ring was, as predicted, invisible; but a careful watch was kept for its reappearance from behind the planet. At $2^h\ 3^m\ 55^s$ the star was just glimpsed at

the edge of the disc; it was better visible fifteen seconds later, and the observed time of distinct separation was

$$2^h\ 5^m\ 30^s{\cdot}7.$$

The observations were made by myself and recorded by Mr Bappu. The disappearance behind the ring was not at all similar to that in the case of a lunar occultation; while the reappearance from behind the ball was almost like that from the moon's bright limb, but not so instantaneous.

The times are all given in Indian Standard Time—0^h commencing at midnight. Add $6^h\ 30^m$ to convert into G.M.T.

Nizamiah Observatory, Hyderabad (Deccan), India:
1920 *April.*

Note on the Proper Motion of B.D. 63° 869 (*Helsingfors* 6113).

By Ethel F. Bellamy.

(*Communicated by Professor H. H. Turner.*)

In forming ledgers of the *Astrographic Catalogue* for the Vatican zones (+64° to +55°), I came across two consecutive stars in the Helsingfors Catalogue, Nos. 6112 and 6113, which gave anomalous results. In the case of 6112 it was at last found that there was a misprint in the declination which should read 63° 12′ 49″·3 instead of 63° 21′ 49″·3.

The next star 6113 seemed to have a considerable P.M., as far as could be judged from comparison of plates taken in 1906 and 1909: and to strengthen the evidence, Mr. Bellamy kindly took another plate of the region in 1914, which, however, was not measured till recently. It amply confirms the P.M. suspected.

The corrections to the Helsingfors place (observed in 1874·2) are as follows, in units of ·001 réseau interval (= 0″·3):—

Δx.	Δy.	Centre. h m	°	Date.
−34	−63	9 44	+63	1906·2
−33	−68	9 54	+64	1909·2
−40	−77	9 44	+63	1914·2

The corrected place of the star for 1900·0 is thus

$$9^h\ 48^m\ 47^s{\cdot}20 \quad 63°\ 16′\ 39″{\cdot}0 \text{ (mag. } 8{\cdot}5)$$

and the P.M. is approximately $-0^s{\cdot}0445$ and $-0″{\cdot}52$, or in arc of a great circle 0″·58, sufficiently large to make a determination of parallax interesting.

I measured in all 77 stars on the 1914 plate. Comparison with the Vatican plate of 1906 revealed no other differences of suspected P.M.

A Mathematical Examination of the Methods of determining the Accuracy of an Observation by the Mean Error, and by the Mean Square Error. By R. A. Fisher, M.A.

1. In estimating the precision of a number of observations two methods are in common use: that of the Mean Square Error, and that of the Mean Error. It is, I believe, usually admitted that the former has the firmer mathematical basis, although it is sometimes asserted that the latter is more accurate. It is not generally recognised that the merits of the two methods may be compared with precision. The case is of interest in itself, and it will be found that the method here outlined is illuminating in all similar cases, where the same quantity may be ascertained by more than one statistical formula.

Suppose the probability distribution of each observation to be centred about a true mean m, with normal distribution and standard deviation σ, so that the chance of any observation falling in the range dx is

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-m)^2}{2\sigma^2}}dx.$$

The unknown, σ, is to be determined from n observations, $x_1, x_2, x_3, \ldots, x_n$. Let

$$n\bar{x} = \mathrm{S}(x),$$

$$n\sigma_1 = \sqrt{\frac{\pi}{2}}\,\mathrm{S}(|x-\bar{x}|),$$

and

$$n\sigma_2^2 = \mathrm{S}(x-\bar{x})^2,$$

then σ_1 is the value obtained by the method of Mean Error, σ_2 that obtained by the method of Mean Square Error, and σ the true value. Both σ_1 and σ_2 may be adjusted by means of appropriate functions of n so as to make the mean value of each of them obtained from a number of samples agree with the true value, but this for the moment is immaterial.

2. The distribution of σ_2.

I have described elsewhere ("Frequency Distribution of the Values of the Correlation Coefficient in Samples from an Indefinitely Large Population," *Biometrika*, **10**, 507) a method by which the frequency distribution of σ_2 may be established. If $x_1, x_2, \ldots, x_n$ are co-ordinates in generalised space of n dimensions, then any sample is represented by a single point having the observed values as co-ordinates. Let O be the origin and C the point at which every observed value is equal to m; then at any point along the line OC, produced indefinitely in both directions, all the co-ordinates are equal.

Let the point P represent the sample; from P draw PM perpendicular to OC, then it is easy to see that M is the point,

$$x_1 = x_2 = x_3 = \ldots x_n = \bar{x},$$

and that PM^2 is

$$S(x-\bar{x})^2 = n\sigma_2{}^2.$$

Now, since the chance of any observation falling in the range dx is

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx,$$

the chance of the sample falling in the space $dx_1, dx_2, \ldots dx_n$ is

$$\frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{S(x-m)^2}{2\sigma^2}} dx_1, dx_2, \ldots dx_n,$$

but

$$S(x-m)^2 = n(\bar{x}-m)^2 + n\sigma_2{}^2,$$

and the element of volume is evidently proportional to

$$d\bar{x} \cdot \sigma_2{}^{n-2} d\sigma_2,$$

so that the chance of a sample falling in the range $d\bar{x}\, d\sigma_2$ is proportional to

$$e^{-\frac{n(\bar{x}-m)^2}{2\sigma^2}} dx \cdot \sigma_2{}^{n-2} \quad e^{-\frac{n\sigma_2{}^2}{2\sigma^2}} d\sigma_2.$$

The distributions of $\bar{x}$ and σ_2 are therefore perfectly independent; the chance of $\bar{x}$ falling in the range $d\bar{x}$ is

$$\frac{\sqrt{n}}{\sigma\sqrt{2\pi}} e^{-\frac{n(\bar{x}-m)^2}{2\sigma^2}} d\bar{x},$$

and the chance of σ_2 falling in the range $d\sigma_2$ is

$$\frac{n^{\frac{1}{2}(n-1)}}{2^{\frac{1}{2}(n-3)} \cdot \left(\frac{n-3}{2}\right)!} \cdot \frac{\sigma_2{}^{n-2} d\sigma_2}{\sigma^{n-1}} \cdot e^{-\frac{n\sigma_2{}^2}{2\sigma^2}}.$$

We may note at once that the mean value of σ_2 is

$$\sqrt{\frac{2}{n}} \cdot \frac{\left(\frac{n-2}{2}\right)!}{\left(\frac{n-3}{2}\right)!} \sigma,$$

while the mean value of $\sigma_2{}^2$ is

$$\frac{n-1}{n} \sigma^2.$$

As n is increased the curve rapidly tends to the normal form; the mean is approximately

$$\left(1 - \frac{3}{4n}\right)\sigma,$$

* The symbol $x!$ is here used in a sense equivalent to $\Pi(x)$ or $\Gamma(x+1)$, whether x is an integer or not.

whence it is easy to see that the standard error of σ_2 is

$$\frac{\sigma}{\sqrt{2n}}.$$

If it were desired to bring the mean into coincidence with the true value σ, the value of σ_2 obtained should evidently be multiplied by

$$\sqrt{\frac{n}{2}}\,\frac{\left(\frac{n-3}{2}\right)!}{\left(\frac{n-2}{2}\right)!}.$$

This correction, if it be so considered, is never of importance; when n is large the value $\frac{3}{4n}$ is of higher order than the standard error. Even when $n=2$, and the correction amounts to an increase of 25 per cent., the standard error is much greater, being 75 per cent. of the mean.

3. Standard error of σ_1.

If the co-ordinates $x_1, x_2, \ldots, x_n$ are the deviations of individuals of a sample from its mean, the representative point lies in a plane space of $(n-1)$ dimensions, in which

$$\mathrm{S}(x)=0.$$

The frequency density at any distance, r, from the origin is proportional to

$$e^{-\frac{r^2}{2\sigma^2}},$$

and

$$r^2=\mathrm{S}(x^2).$$

The region in which any co-ordinate has an assigned value, x_1, is a plane space of $(n-2)$ dimensions, at a distance

$$x_1\sqrt{\frac{n}{n-1}}$$

from the origin, and the frequency with which x_1 falls into the range dx_1 is therefore proportional to

$$e^{-\frac{nx_1^2}{2(n-1)\sigma^2}}dx_1.$$

Thus deviations from the mean of samples of n of a normal population are themselves normally distributed. The deviations from the mean of the population are, however, independent, while deviations from the mean of the same sample are not. Consider the distribution of pairs of values, x_1 and x_2.

The space in which the representative points lie is parallel to the line

$$x_1+x_2=0$$
$$x_3=x_4=\ldots=x_n=0,$$

while it makes with the line

$$x_1 = x_2$$
$$x_3 = x_4 = \ldots = x_n = 0$$

an angle the cosine of which is

$$\sqrt{\frac{n-2}{n}}.$$

Consequently the frequency in the range $dx_1 dx_2$ is proportional to

$$e^{-\frac{1}{2\sigma^2}\left\{\frac{(x_1-x_2)^2}{2}+\frac{n}{n-2}\cdot\frac{(x_1+x_2)^2}{2}\right\}}dx_1 dx_2$$
$$= e^{-\frac{n-1}{2(n-2)\sigma^2}\left\{x_1^2+\frac{2x_1x_2}{n-1}+x_2^2\right\}}dx_1 dx_2,$$

showing a surface of normal correlation, with correlation coefficient, $-\frac{1}{n-1}$, between any two deviations.

If the deviation be considered without reference to sign, each is distributed from 0 to ∞ with frequency,

$$\sqrt{\frac{2n}{\pi(n-1)}}e^{-\frac{nx^2}{2(n-1)\sigma^2}}\cdot\frac{dx}{\sigma},$$

and each pair with frequency,

$$\frac{2(n-1)}{\pi\sigma^2\sqrt{n(n-2)}}e^{-\frac{n-1}{2(n-2)\sigma^2}(x_1^2+x_2^2)}\cosh\frac{(x_1x_2)}{(n-2)\sigma^2}dx_1dx_2.$$

The mean value of x is therefore

$$\sigma\sqrt{\frac{n-1}{n}}\cdot\sqrt{\frac{2}{\pi}},$$

so that the mean value of σ_1 is

$$\sigma\sqrt{\frac{n-1}{n}}$$

as is generally known.

The mean value of x^2 is

$$\frac{n-1}{n}\sigma^2,$$

and that of xy is

$$\frac{2\sigma^2}{\pi n}\left(\sqrt{n(n-2)}+\sin^{-1}\frac{1}{n-1}\right),$$

whence it follows that the mean value of σ_1^2 is

$$\frac{(n-1)\sigma^2}{n^2}\left(\frac{\pi}{2}+\sqrt{n(n-2)}+\sin^{-1}\frac{1}{n-1}\right).$$

As in the case of σ_2, the mean differs from the true value by a term of the order $\frac{1}{n}$. When n is large this is insignificant, for the mean value of σ_1^2 is approximately

$$\frac{n-1}{n}\sigma^2\left\{1+\left(\frac{\pi}{2}-1\right)\frac{1}{n}\right\};$$

hence the standard error is

$$\frac{\sigma}{\sqrt{n}}\sqrt{\pi-2}.$$

As n is made large, therefore the standard error of σ_1 tends to bear a constant ratio to that of σ_2. The former is the larger in the ratio $\sqrt{\pi-2}$; in other words, the value of the standard deviation, or probable error, obtained from the Mean Square Deviation of a sample has greater weight by 14 per cent. than that obtained from the Mean Deviation. To obtain a result of equal accuracy by the latter method, the number of observations must be increased by 14 per cent.*

4. The derivate of minimum error.

The correlation between deviations, taken positive, being of order $\frac{1}{n^2}$, does not, as we have seen, affect the expression for the variance of a derivate, when n is large, because this is of order $\frac{1}{n}$. We may, then, in examining the comparative variance of different derivates ignore this correlation, and treat the deviations as though they were independent. The term "variance" is used here as elsewhere to signify the square of the standard deviation or standard error; by the "relative variance" is intended the same quantity divided by the square of the mean.

It is easy to verify that, although the variance is diminished as we pass from the derivate of the first power to that of the second, it is increased as we pass from the second power to the third. It is, therefore, of interest to determine for what power it is actually a minimum.

If $\sigma^p\mu_p$ is the mean value of x^p, then

$$\mu_p=\sqrt{\frac{2}{\pi}}\int_0^\infty t^p e^{-\frac{1}{2}t^2}dt=\left(\frac{p-1}{2}\right)!\,\frac{2^{\frac{1}{2}p}}{\sqrt{\pi}},$$

* Mr. Fisher kindly allows me to correct here an erroneous statement in my book, *Stellar Movements*, p. 147, footnote. I think it accords with the general experience of astronomers that, for the errors commonly occurring in practice, the mean error is a safer criterion of accuracy than the mean square error, especially if any doubtful observations have been rejected; but I was wrong in claiming a theoretical advantage for the mean error in the case of a truly Gaussian distribution. My formulæ were somewhat different from Mr. Fisher's, since I considered the deviations of σ_2 from σ instead of from $\bar{\sigma}_2$; but, as he points out, this correction (as I considered it) is of minor importance, and my mistake arose in the numerical evaluation of the results. —A. S. Eddington.

if, then, σ_p is the derivate of the p^{th} power, we take as definition

$$\sigma_p{}^p = \frac{S(x^p)}{n\mu_p}.$$

It is well known that the variance of the p^{th} moment of a sample of n is

$$\frac{1}{n}\left(\mu_{2p} - \mu_p{}^2\right)$$

when n is large; whereas it follows that the relative variance of $\sigma_p{}^p$ is

$$\frac{1}{n}\left(\frac{\mu_{2p}}{\mu_p{}^2} - 1\right),$$

and, therefore, that the relative variance of σ_p is

$$\frac{1}{np^2}\left(\frac{\mu_{2p}}{\mu_p{}^2} - 1\right) = \frac{1}{np^2}\left\{\frac{(p-\frac{1}{2})!}{(\frac{1}{2}p-\frac{1}{2})!^2}\sqrt{\pi} - 1\right\}.$$

Putting $p = 1$, 2, and 3, we have $\frac{\pi-2}{2n}$, $\frac{1}{2n}$, and $\frac{15\pi-8}{72n}$, being in the ratio 1·1416 : 1 : 1·0868.

If the relative variance is a minimum for variations of p, then must

$$\frac{2}{p}\left\{\frac{(p-\frac{1}{2})!}{(\frac{1}{2}p-\frac{1}{2})!^2}\sqrt{\pi} - 1\right\} = \frac{(p-\frac{1}{2})!\sqrt{\pi}}{(\frac{1}{2}p-\frac{1}{2})!^2}\left\{\frac{d}{dp}\log(p-\tfrac{1}{2})! - \frac{d}{d(\frac{1}{2}p)}\log(\tfrac{1}{2}p-\tfrac{1}{2})!\right\}.$$

Now, when $p = 2$, the factor outside the right-hand bracket reduces to 3, and the left-hand side of the equation is therefore 2. The right-hand bracket may be evaluated from the definite integral,

$$\frac{d}{dz}\log(z!) = \int_0^\infty \left(\frac{1}{t} - \frac{e^{-tz}}{1-e^{-t}}\right)e^{-t}dt,$$

writing successively $1\frac{1}{2}$ and $\frac{1}{2}$ for z, as in the bracket, there remains

$$\int_0^\infty \left(e^{-\frac{1}{2}t} - e^{-\frac{3}{2}t}\right)\frac{e^{-t}}{1-e^{-t}}dt$$

$$= \int_0^\infty e^{-\frac{3}{2}t}dt = \frac{2}{3}.$$

The equation is therefore satisfied, the relative variance having its minimum value when p is 2. The mean square deviation is the derivate with the minimum relative variance.

5. The distribution of pairs of values of σ_1 and σ_2, when $n = 4$.

Full knowledge of the effects of using one rather than another of two derivates can only be obtained from the frequency surface of pairs of values of the two derivates. By integration with respect to one derivate or the other, the two frequency curves can be obtained and compared, in respect of any quality which may

be in question; but the additional information supplied by the mutual frequency surface is essential to a thorough examination of the question.

When n is large the problem is simplified by the fact that both curves rapidly approach the normal form centred about the true value as mean; the only possible difference between such curves is in the standard deviation. For small values of n the case is much more complicated; failing a complete expression for the frequency surface of σ_1 and σ_2 in terms of n, it will be best to investigate this surface in the single case, $n = 4$. This single case will be found sufficient to bring out the decisive features of the general surface.

If x_1, x_2, x_3, and x_4 are the four observations, then the chance of all four observations falling into their respective elementary ranges is

$$\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^4 e^{-\frac{\mathrm{S}(x-m)^2}{2\sigma^2}}\, dx_1\, dx_2\, dx_3\, dx_4.$$

As in Article 2, this frequency density is the product of two factors, one depending only on $\bar{x}$,

$$\frac{\sqrt{n}}{\sigma\sqrt{2\pi}}\, e^{-\frac{n(\bar{x}-m)^2}{2\sigma^2}}\, d\bar{x},$$

and the other on σ_2,

$$\frac{1}{(\sigma\sqrt{2\pi})^3}\, e^{-2\frac{\sigma_2^2}{\sigma^2}}\, dv,$$

in which dv stands for the element of volume in the plane three-dimensional space,

$$\mathrm{S}(x-\bar{x}) = 0.$$

Within this space the values of $(x-\bar{x})$ will be positive or negative according as the representative point lies on one side or the other of four planes, through M, drawn parallel to the faces of a regular tetrahedron. The surface of a sphere with M as centre is therefore divided into 14 areas (all combinations of sign being possible except *all positive* and *all negative*). Of these areas 6 are regular four-sided figures, including the cases in which two deviations are positive and two negative; the remaining 8, regular three-sided figures, include the cases in which one deviation is of opposite sign to the other three. All the sides of these figures are 60° and the angles $\cos^{-1} \pm \frac{1}{3}$. The fraction of the total area included in the four-sided figures is

$$3 - \frac{6}{\pi}\cos^{-1}\tfrac{1}{3} = \cdot 6490,$$

and the remainder in the three-sided areas is

$$\frac{6}{\pi}\cos^{-1}\tfrac{1}{3} - 2 = \cdot 3510.$$

The distribution of σ_1 is different in these two groups of regions.

When two of the deviations are positive and two negative, σ_1 will be constant on a plane space of the type,

$$(x_1-\bar{x})+(x_2-\bar{x})-(x_3-\bar{x})-(x_4-\bar{x})=\sqrt{\frac{2}{\pi}}\cdot 4\sigma_1.$$

This space is at right angles to that which we have considered, namely

$$S(x-\bar{x})=0,$$

and its least distance from the centre therefore lies in that space. This distance is $\sigma_1\sqrt{\frac{8}{\pi}}$, and passes through the centre of figure of the spherical quadrangle in which the representative point lies. If θ be the angular distance of this point from the centre of figure

$$2\sigma_2\cos\theta=\sigma_1\sqrt{\frac{8}{\pi}},$$

i.e.

$$\cos\theta=\sqrt{\frac{2}{\pi}}\cdot\frac{\sigma_1}{\sigma_2} \quad . \quad . \quad . \quad . \quad \text{Ia.}$$

In the triangular regions the plane space over which σ_1 is constant, such as

$$(x_1-\bar{x})+(x_2-\bar{x})+(x_3-\bar{x})-(x_4-\bar{x})=\sqrt{\frac{2}{\pi}}\cdot 4\sigma_1,$$

is not perpendicular to the space in which the representative point lies, but makes with it an angle of 60°, the distance from the centre to the plane of intersection is therefore

$$\sqrt{\frac{8}{\pi}}\sigma_1\operatorname{cosec}60^\circ=\sigma_1\sqrt{\frac{32}{3\pi}}$$

whence

$$\cos\theta=\sqrt{\frac{8}{3\pi}}\cdot\frac{\sigma_1}{\sigma_2} \quad . \quad . \quad . \quad . \quad \text{Ib.}$$

The frequency distribution of σ_1, for a given value of σ_2, is thus reduced to the frequency of occurrence of different values of θ, in two types of spherical figures. For the quadrangles the greatest possible value of θ is 45°, while the least distance from the perimeter to the centre is $\sin^{-1}\sqrt{\frac{1}{3}}$. From these 6 regions we have

$$\left.\begin{array}{ll}\text{From } 0 \text{ to } \sin^{-1}\dfrac{1}{\sqrt{3}} & \text{frequency } 3\sin\theta d\theta \\ \text{From } \sin^{-1}\dfrac{1}{\sqrt{3}} \text{ to } 45^\circ & \text{frequency } 3\sin\theta\left(1-\dfrac{4}{\pi}\cos^{-1}\dfrac{1}{\sqrt{2}\tan\theta}\right)d\theta\end{array}\right\} \text{IIa.}$$

The greatest value of θ in the triangles is $\sin^{-1}\frac{1}{\sqrt{3}}$, and the least distance from the perimeter to the centre is $\sin^{-1}\frac{1}{3}$, therefore from 8 such regions we have

$$\left.\begin{array}{lll} \text{from } 0 \text{ to } \sin^{-1}\frac{1}{3} & \text{frequency} & 4\sin\theta d\theta \\ \text{from } \sin^{-1}\frac{1}{3} \text{ to } \sin^{-1}\frac{1}{\sqrt{3}} & \text{frequency} & 4\sin\theta\left(1-\frac{3}{\pi}\cos^{-1}\frac{1}{\sqrt{8}\tan\theta}\right)d\theta \end{array}\right\} \text{IIB.}$$

Substituting for θ in expression II., we obtain for the distribution of σ_1 for a given value of σ_2,

$$\left.\begin{array}{lll} \text{from } \sigma_2\sqrt{\frac{\pi}{2}} \text{ to } \sigma_2\sqrt{\frac{\pi}{3}} & \text{frequency} & 3\sqrt{\frac{2}{\pi}}\frac{d\sigma_1}{\sigma_2} \\ \text{from } \sigma_2\sqrt{\frac{\pi}{3}} \text{ to } \sigma_2\sqrt{\frac{\pi}{4}} & \text{frequency} & 3\sqrt{\frac{2}{\pi}}\left(1-\frac{4}{\pi}\cos^{-1}\frac{\sigma_1}{\sqrt{\pi\sigma_2^2-2\sigma_1^2}}\right)\frac{d\sigma_1}{\sigma_2} \end{array}\right\} \text{IIIA.}$$

for the quadrangles, and for the triangles

$$\left.\begin{array}{lll} \text{from } \sigma_2\sqrt{\frac{3\pi}{8}} \text{ to } \sigma_2\sqrt{\frac{\pi}{3}} & \text{frequency} & 8\sqrt{\frac{2}{3\pi}}\frac{d\sigma_1}{\sigma_2} \\ \text{from } \sigma_2\sqrt{\frac{\pi}{3}} \text{ to } \sigma_2\sqrt{\frac{\pi}{4}} & \text{frequency} & 8\sqrt{\frac{2}{3\pi}}\left(1-\frac{3}{\pi}\cos^{-1}\frac{\sigma_1}{\sqrt{3\pi\sigma_2^2-8\sigma_1^2}}\right)\frac{d\sigma_1}{\sigma_2} \end{array}\right\} \text{IIIB.}$$

These two frequency curves are shown on opposite sides of the same base in fig. 1. The existence of two or more distinct curves according to the partition of the observations by the mean, and the existence of one or more discontinuities are no doubt characteristic of such curves in general. For higher values of n, the right-hand side would no longer be truncated, but would meet the base with increasingly high contact.

From the expressions III., may be obtained the frequency surface. For the frequency in the range $d\sigma_2$, being, by Article 2,

$$\frac{8\sqrt{2}}{\sqrt{\pi}}\cdot\frac{\sigma_2^2 d\sigma_2}{\sigma^3}e^{-2\frac{\sigma_2^2}{\sigma^2}}d\sigma_2,$$

the frequency in the range $d\sigma_1\,d\sigma_2$ is

$$\left.\begin{array}{l} \sqrt{\frac{\pi}{2}}>\frac{\sigma_1}{\sigma_2}>\sqrt{\frac{\pi}{3}} \quad \frac{48}{\pi}e^{-\frac{2\sigma_2^2}{\sigma^2}}\frac{\sigma_2 d\sigma_1 d\sigma_2}{\sigma^3} \\ \sqrt{\frac{\pi}{3}}>\frac{\sigma_1}{\sigma_2}>\sqrt{\frac{\pi}{4}} \quad \frac{48}{\pi}e^{-\frac{2\sigma_2^2}{\sigma^2}}\left(1-\frac{4}{\pi}\cos^{-1}\frac{\sigma_1}{\sqrt{\pi\sigma_2^2-2\sigma_1^2}}\right)\cdot\frac{\sigma_2 d\sigma_1 d\sigma_2}{\sigma^3} \end{array}\right\} \text{IVA.}$$

when two observations lie on each side of the mean, and

$$\left.\begin{aligned}\sqrt{\frac{3}{8}\pi} > \frac{\sigma_1}{\sigma_2} > \sqrt{\frac{1}{3}\pi} \quad & \frac{128}{\pi\sqrt{3}}\, e^{-\frac{2\sigma_2^2}{\sigma^2}} \cdot \frac{\sigma_2}{\sigma_3}\, d\sigma_1 d\sigma_2 \\ \sqrt{\frac{1}{3}\pi} > \frac{\sigma_1}{\sigma_2} > \sqrt{\frac{1}{4}\pi} \quad & \frac{128}{\pi\sqrt{3}}\, e^{-\frac{2\sigma_2^2}{\sigma^2}} \left(1 - \frac{3}{\pi}\cos^{-1}\frac{\sigma_1}{\sqrt{3\pi\sigma_2^2 - 8\sigma_1^2}}\right)\frac{\sigma_2}{\sigma^3}\, d\sigma_1 d\sigma_2\end{aligned}\right\} \text{IVB.}$$

when three are on one side and one on the other.

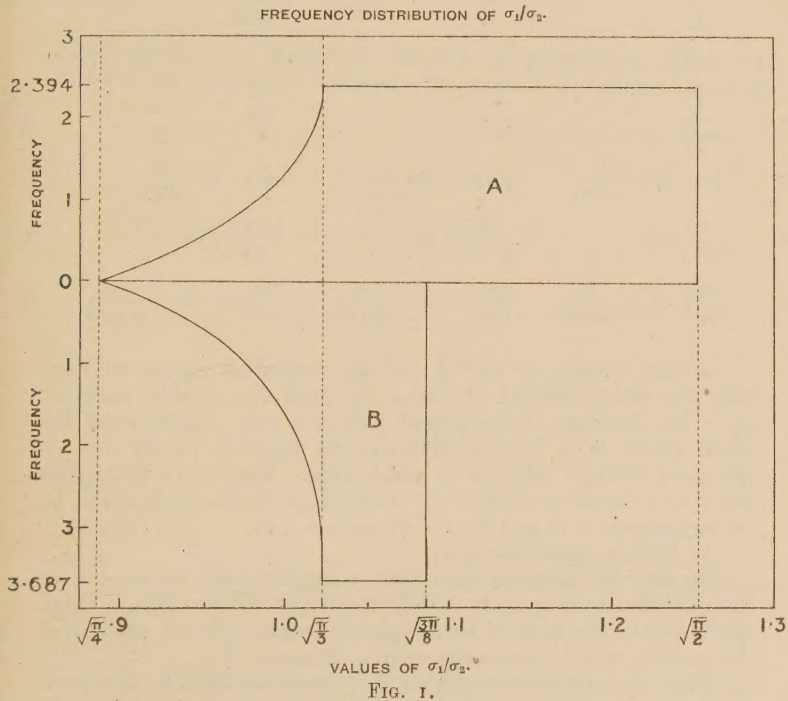


FIG. 1.

In expressions IV. we have the complete account of the facts of random sampling respecting the two variables σ_1 and σ_2; it will be of interest first to obtain the frequency curves of σ_1.

6. The distribution of σ_1.

The expressions IV. may be integrated with respect to σ_2, over its whole range of variability from 0 to ∞; by so doing we arrive at two frequency curves corresponding to the two partitions of the observations,

$$\frac{12}{\pi}\, e^{-\frac{4\sigma_1^2}{\pi\sigma^2}} \left\{1 - \frac{4}{\pi}\, e^{-\frac{2\sigma_1^2}{\pi\sigma^2}} \int_0^1 e^{-\frac{2\sigma_1^2 t^2}{\pi\sigma^2}} \cdot \frac{dt}{t^2+1}\right\} \frac{d\sigma_1}{\sigma} \quad \text{VA.}$$

and

$$\frac{32}{\pi\sqrt{3}}\, e^{-\frac{16\sigma_1^2}{3\pi\sigma^2}} \left\{1 - \frac{3\sqrt{3}}{\pi}\, e^{-\frac{2\sigma_1^2}{3\pi\sigma^2}} \int_0^1 e^{-\frac{2\sigma_1^2 t^2}{\pi\sigma^2}} \cdot \frac{dt}{3t^2+1}\right\} \frac{d\sigma_1}{\sigma} \quad \text{VB.}$$

Of these curves it is easy to calculate the moments, and thence to find those of the compound curve obtained by throwing them together, as is done if we consider the distribution of σ_1 without regard for the manner in which the observations are parted by the mean.

The moments of these three curves and of the corresponding curve for σ_2 are shown in the following table:—

	σ_1. A.	σ_1. B.	σ_1. Total.	σ_2.
μ_0	$3-\frac{6a}{\pi}$	$\frac{6a}{\pi}-2$	1	1
μ_1/σ	$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{\pi}}$
μ_2/σ^2	$\frac{3}{8}(\pi-2a)+\frac{1}{2\sqrt{2}}$	$\frac{3}{16}(3a-\pi)+\frac{1}{4\sqrt{2}}$	$\frac{3}{16}(\pi-a)+\frac{3}{4\sqrt{2}}$	$\frac{3}{4}$
μ_3/σ^3	$\frac{\pi}{3\sqrt{3}}+\frac{1}{8}$	$\frac{13\pi}{96\sqrt{3}}+\frac{1}{16}$	$\frac{5\pi\sqrt{3}}{32}+\frac{3}{16}$	$\sqrt{\frac{2}{\pi}}$
μ_4/σ^4	$\frac{9\pi}{64}(\pi-2a)+\frac{29}{64\sqrt{2}}$	$\frac{27\pi}{512}(3a-\pi)+\frac{49}{384\sqrt{2}}$	$\frac{9\pi}{512}(5\pi-7a)+\frac{133}{384\sqrt{2}}$	$\frac{15}{16}$

a here stands for $\cos^{-1}\frac{1}{3}$; of the numerical values we need only cite the coefficient of variation, ·4296 for σ_1 and ·4220 for σ_2. The derivate of the second power is less variable even for small values of n, but the difference in weight in favour of σ_2 is increased fourfold when n is made large. The curve for σ_1 has not only a larger coefficient of variation, it is also more skew, β_1 is ·297 against ·238 and β_2 is 3·28 against 3·11.

7. Unique properties of σ_2.

So far the variables have been compared only in respect of the quantitative characters of their frequency distributions. There exists also in the form of the frequency surface (IV.) a qualitative distinction, which reveals the unique character of σ_2.

From the manner in which the frequency surface has been derived, as in expressions III., it is evident that:—

For a given value of σ_2, the distribution of σ_1 is independent of σ.

On the other hand, it is clear from expressions (IV.) and (V.) that for a given value of σ_1 the distribution of σ_2 does involve σ. In other words, if, in seeking information as to the value of σ, we first determine σ_1, then we can still further improve our estimate by determining σ_2; but if we had first determined σ_2, the frequency curve for σ_1 being entirely independent of σ, the actual value of σ_1 can give us no further information as to the value of σ. The whole of the information to be obtained from σ_1 is included in that supplied by a knowledge of σ_2.

This remarkable property of σ_2, as the methods which we have used to determine the frequency surface demonstrate, follows from the distribution of frequency density in concentric spheres

over each of which σ_2 is constant. It therefore holds equally if σ_3 or any other derivate be substituted for σ_1. If this is so, then it must be admitted that:—

The whole of the information respecting σ, *which a sample provides, is summed up in the value of* σ_2.

This unique superiority of σ_2 is dependent on the form of the normal curve,

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} dx,$$

which leads to a frequency density in generalised space distributed on concentric spheres. Since it is sometimes urged in

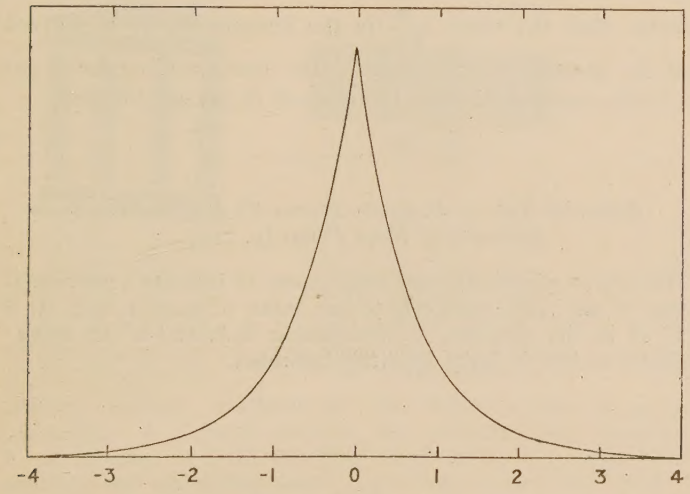


FIG. 2.

favour of the Mean Error that it gives less weight to the large deviations, and that these large deviations do in fact occur in excess of the normal expectation, it is of interest to see if any curve is related to the Mean Error in the same way as is the normal curve to the Mean Square Error.

The somewhat artificial curve

$$\frac{1}{\sigma\sqrt{2}} e^{-\frac{|x-m|}{\sigma}\sqrt{2}} dx$$

replaces the generalised spheres by generalised octahedra, upon the surfaces of which σ_1 is constant, provided $\bar{x} = m$. For large values of n this condition is sufficiently approached and σ_1 may be taken as the ideal measure of σ for curves of this type. When n is small, and allowance has to be made for the aberrations of $\bar{x}$, the figure on which σ_1 is constant is the central section of a

generalised octahedron, as was seen in the case $n=4$, where the figure over which σ_1 is constant was found to be bounded by six squares and eight equilateral triangles; while the surface of equal probability is in general an eccentric section in which the squares become rectangles and the triangles are not all equal.

When n is large, however, it does not seem unreasonable to employ σ_1 to samples from curves which resemble the above rather than the normal curve. The value of β_2 (the ratio of the fourth moment to the square of the second moment) seems well fitted to provide a test. If this is near to 3 the Mean Square Error will be required; if, on the other hand, it approaches 6, its value for the double exponential curve, it may be that σ_1 is a more suitable measure of dispersion. It should not be forgotten, however, that the factor $\sqrt{\frac{\pi}{2}}$ in the formula for σ_1 is derived from the normal curve of errors. The corresponding factor for the double exponential curve is $\sqrt{2}$, about 12 per cent. bigger.

Editorial Note on Professor Barnard's Micrometrical Measures of Nova Persei (p. 710).

The above micrometer measures appear to indicate a centennial motion of the nova relatively to the mean of stars 1, 2, 3, 5, 8 of $1''\cdot08$ in the direction of diminishing R.A. and $2''\cdot20$ south; the latter motion is fairly definitely indicated.